**The Shape and Space Strand: Outcome SS7.1**

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| **Outcome** | **Indicators** |
| SS7.1 Demonstrate an understanding of circles including circumference and central angles.  [C, CN, R, V]  *In support of the K-12 Mathematics goals of Spatial Sense, Number Sense, Logical Thinking and Mathematical Attitude.* | 1. Identify the characteristics of a circle. 2. Define and illustrate the relationship between the diameter and radius of a circle. 3. Answer the question “how many radii does a circle have and why?” 4. Answer the question “how many diameters does a circle have and why?” 5. Explain, with illustrations, why a specified point and radius length (or diameter length) describes exactly one circle. 6. Illustrate and explain that the diameter is twice the radius in a circle. 7. Generalize from investigations the relationship between the circumference and the diameter of a circle/ 8. Define pi () and explain how it is related to circles. 9. Demonstrate that the sum of the central angles of a circle is 360°. 10. Sort a set of angles as central angles of a circle or not. 11. Draw a circle with a specific radius or diameter with and without a compass. 12. Solve problems involving circles. |
| **Learning Space** [**Top**](#top) | |
| In grades 5 and 6, students have been considering various 2-D polygons in terms of their properties, perimeter and areas. In grade 7, the students move to the non-polygonal shape of a circle. In this outcome, the students explore the characteristics and properties of circles as well as its circumference. This study also leads them to their first encounter with an irrational number. In grade 8 the students continue this study of irrational numbers when they begin to explore square roots.  The circle adds two new dimensions to the students’ study of 2-D shapes: shapes with non-linear sides and the existence of irrational numbers. Both of these concepts may seem obvious to the teacher, but for the students they are often confusing. Thus, students often do not develop an understanding of either concept, but rather resort to memorization of terms and procedures. In order to prevent this from happening, it is important that sufficient time and hands-on exploration be incorporated for the students’ learning activities.  Physical Education and Arts Education both provide contexts in which the students can explore the properties of circles and of the number. Having the students explore movement within a circle, or finding ways to represent can benefit the students’ development of deep understandings of circles. | |

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| **What Students Should…** [**Top**](#top) | | |
| **Know**   * The terms radius, diameter, circumference, central angle, and pi. | **Understand**   * That a circle has an infinite number of radii and diameters, all of equal length. * That is the symbol that is used to represent the irrational number defined by the quotient of the circumference of a circle divided by its diameter length. * That circumference is a linear measurement despite it being measured on a curve. * That circumference is the perimeter of a circle. * That the sum of the central angles in a circle is 360°. * That a circle is the set of all points that are the same distance from a fixed point in two-dimensions. | **Be Able to Do**   * Determine the length of the radius or diameter of a circle given the length of the other measurement. * Determine the circumference of a circle given its diameter or radius. * Determine the measure of an angle within a circle. * Draw circles using a variety of materials. * Solve problems involving diameter, radius, circumference and/or central angles. |
| **Key Questions** [**Top**](#top) | | |
| * What makes circles different from other 2-D shapes you have studied? In what ways are they the same? * Why is circumference a one-dimensional measurement? * What is? | | |
| **S****uggestions for Assessment:** [**Top**](#top) | | |
| **Big Idea:**  Properties of circles.  **Suggestions for assessment tasks:**   1. Ask the students to construct a circle with a given radius. Keep a record of the method used. Ask the students if they could have constructed it in a different way and to describe the method. 2. Give the students a circle and ask them to construct an acute central angle and explain how they know that it is a central angle and how they know it is acute. Have the students determine the measure of the angle. 3. Give the students a diagram in which they can determine the measure of a central angle without measuring. 4. Have the students identify a radius in a circle that is a face of a 3-D object and then construct and determine the length of a radius and a diameter for the circle. Question the students about the strategies they used for determining the two lengths. Note whether they know the relationship between the radius and diameter.   **What to look for:**   * See *Properties of Circles Rubric.* * See *Properties of Circles Anecdotal Record Sheet* | | |
| **Big Idea:**  - an irrational number.  **Suggestions for assessment tasks:**   1. Provide the students with a cylinder and ask them to determine the circumference of the base of the cylinder. Observe the students’ methods and strategies. If they are using concrete methods to determine the circumference, ask the students to explain and demonstrate how they could determine the circumference without doing the actual measuring. 2. Have the students create a story, poster, drama, painting etc. in which they explain what  is, and how it is related to circles. As a class, design a rubric or rating scale for assessing the students’ creations. There are many examples on the internet of artists’ renditions of .   **What to look for:**   * See - *an Irrational Number Rubric.* | | |
| **S****uggestions for Instruction:** [**Top**](#top) | | |
| **Big Idea:**  Properties of circles.  **Suggestions for instructional activities**   1. Provide the students with a number of 3-D objects that have at least one face that is a circle and pictures of 2-D circles. Ask the students to determine the longest distance across the circle and to show at least three lines that are that long and stretch across the entire circle. The students can work in pairs, each one checking to make sure that a longer line isn’t possible. 2. As a class, discuss the lines that they drew and anything that they noticed. They should notice that the lines all cross (intersect) at one point in the very centre of the circle. If their sketches weren’t exact enough to get this result, have the students describe their strategies for placing these longest lines across the circle and follow their instructions on a circle shown on the board or on the overhead. (Dynamic geometric software can also be used to demonstrate the intersection of the diameters at the centre of the circle). Define the point of intersection as the centre of the circle (or centre point) and define the lines as diameters of the circle. | | |

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| 1. Show the students how to use a compass to construct a circle if they don’t already know how. As a homework task, ask the students to think about and explore the following questions: “Why does a compass give a circle and not a different 2-D shape?” and “How else could you construct a circle?” Have the students discuss their answers and questions in the next class. Develop the idea of a circle being all the points that are the same distance from the centre of the circle. Define this distance as being the length of a radius, and define radius as a line segment joining the centre of a circle and any point on the outside of the circle. Have the students explore the question “Do other 2-D shapes have a radius?” Relate the results of their exploration and their alternate strategies to the idea of one distance from the centre of the circle (radius length) to the outside edge of the circle. Students may wish to demonstrate or try out some of the alternate approaches they have seen or heard about. Finally, have the students write about the construction of circles in their journals as well as defining radius and diameter and describing how the two are related to each other. 2. Provide the students with a drawing of a circle. Ask the students to find a line of symmetry for the circle using any strategy they wish. Ask the students what they think the line of symmetry defines and have them share their strategies for creating the line of symmetry. Next, ask the students to explore how they could use lines of symmetry to construct a radius for the circle, diameters for the circle, and how they could locate the centre of the circle. 3. Do a concept attainment activity with the students to develop the concept of a central angle in a circle. Categorize a number of angles shown in a circle as either a YES (it is a central angle) or a NO (it is not a central angle). Have the students individually record (without discussion) what they think are the defining characteristics of the concept. Provide the students with new diagrams and ask them to show (by using thumbs up for YES, thumbs down for NO and thumbs to the side for not sure) where they would place the new diagram. Allow the students to revisit any of the diagrams that have been categorized as not sure, and provide any new YES or NO examples to help clarify any confusion or missing knowledge. Gradually invite the students to share their thinking about the characteristics of the concept and discuss these characteristics as a class, specifically referring to the examples and non-examples that have been categorized. End the activity by defining the YES category as representing central angles of a circle and have the students write a class definition of a central angle in a circle. 4. Have the students use the lines of symmetry previously folded to identify those fold lines that resulted in dividing the circle into 1/4s. Next, have the students discuss what the measure of each of the four central angles must be and why. They may wish to measure using a protractor to be sure. Ask them to compare their results and then find the total measure of the central angles for their circle. Have the students compare their results. It is important that the students recognize that the size of the circle does not impact the sum of the measures of the central angles. 5. Provide the students with additional circles (or they can draw them) and ask them to draw at least two radii in each circle. Have the students measure the central angles formed and then hypothesize a conclusion for the statement “The sum of the measures of the central angles in a circle is \_\_\_\_”. Have the students compare their results and then write a class statement regarding the sum of the measures of the central angles in a circle. Often, students who experiment with large numbers of angles will get slightly more than or slightly less than 360° for this sum. This is because of rounding of the measures of the angles. Have the students work together and explore this occurrence. 6. Give the students a number of problems that require them to determine central angles in circles given measurements of other central angles.   **Big Idea:**  - an irrational number.  **Suggestions for instructional activities**   1. Provide pairs of students with a number of 3-D objects with at least one circular face and give them the task of completing a chart in which they are to record the name of the 3-D object, distance around the circle, the length of the diameter, and the quotient of the distance around divided by the diameter. Have the groups determine the average value for the quotient and record it in a class table. Have the class discuss the results, comparing the values that they attained. Ask the students if there are any outliers in the results. If so, have the group share its entire table and look for anomalies in their records. Have another pair of students re-measure and calculate (note: calculators should be allowed for this task because the focus should be on discovering the spatial relationship, not on long-hand calculations) for the object again to see if they get a different result. In the end, summarize by discussing how this type of process was initially used in the discovery of the number . Have the students explore the value of  on their calculators. 2. Have the students research the number. There are many excellent online resources. You may wish to give them key questions such as: “When was  discovered and by whom?”, “How is  different from other numbers they know, and what is this type of number called?”, “How many decimal places have they found for  so far?”, “What are some approximate values for ?”, and “What do you think about this number?”. 3. Have the students look at their tables from 8 and ask them to write an equation showing the pattern and relationship in the table. (e.g., ) Next, show the students the form of the equation most often seen, and ask them how the two equations are related. Similarly, show the students the radius form of this equation: and ask the students how it is related to the other equations. 4. Provide the students with another 3-D object with a circular face and tell them that they can only measure the radius or diameter and using that measurement they are to determine the circumference of the circle. Have the students share their strategies for determining the radius, diameter and circumference of the circular face. 5. Have the students solve problems in which they need to determine the circumference, diameter, or radius. Challenge the students to solve the problems in different ways and using different forms of the equation. 6. Ask the students to write a problem related to the circumference of an everyday object. Have the students share and solve each others’ problems. |

***Properties of Circles Anecdotal Record***

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| **Construction of Circle** | **Strategy Used:** |
| **Alternate Strategy:** |
| **Central Angles** | **Sketch** |
| **Measuring** |
| **Determining Measure** |
| **Radius and Diameter** | **Identification of a radius** |
| **Construction of a radius** |
| **Construction of a diameter** |