**The Number Strand: Outcome N7.5**

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| **Outcome** | **Indicators** | | |
| N7.5 Develop and demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).  [C, CN, ME, PS, R, V]  *In support of the K-12 Mathematics goals of Number Sense, Spatial Sense and Logical Thinking and Mathematical Attitude.* | * Estimate the sum or difference of positive fractions and/or mixed numbers and explain the reasoning. * Model addition and subtraction of a positive fractions and/or mixed numbers using concrete or visual representations, and record the process used symbolically. * Determine the sum or difference of two positive fractions or mixed numbers with like denominators and explain the strategy used. * Explain how common denominators for fractions and/or mixed numbers and factors are related. * Explain how a common denominator can help when adding fractions and/or mixed numbers. * Determine the sum or difference of two positive fractions or mixed numbers with unlike denominators and explain the strategy used. * Simplify a positive fraction or mixed number by identifying the common factor between the numerator and denominator. * Generalize personal strategies for determining the sum or difference of positive fractions and/or mixed numbers. * Solve a problem involving the addition or subtraction of positive fractions or mixed numbers. * Explain how the sum or difference of positive fractions and/or mixed numbers can be represented symbolically in different ways. | | |
| **Learning Space**  [**Top**](#top) | | | |
| Prior to grade 7, students have not explored or learned about addition and subtraction of fractions. Therefore, it is crucial that their experiences be connected to prior knowledge about the operations of addition and subtraction and about fractions. Moreover, in order to understand rather than memorize how to add and subtract fractions it is very important that students explore the concept pictorially and concretely before moving to an abstract manipulation of the symbolic notation. If when learning about this outcome students have already engaged in outcome N7.3, “Demonstrate an understanding of the relationships between positive decimals, positive fractions (including mixed numbers, proper fractions and improper fractions), and whole numbers”, students can also be engaged in connecting their understanding of adding and subtracting decimals to that of adding and subtracting fractions as a means of determining if the answers they are producing are reasonable.  It is very important that adding and subtracting fractions not be taught as distinct from any adding or subtracting the students have done previously. In order to add or subtract things, they have always had to be the same type of things. The old adage that “you can’t add apples and oranges” applies well here because of the argument that “you can call them fruit and then add them.” We do this same thing when we add fractions. We can’t add fractions with different denominators unless we find a way to “rename” the fractions so that they are the same type of fraction (i.e., same denominators or more importantly, same number of divisions in a whole). This is no different from the regrouping that students have experienced in addition and especially subtraction of whole numbers and decimals. Students will understand fraction addition and subtraction better if they can see logical connections between what they are doing and things that they have done before. | | | |
| It is also crucial that when students have had some initial experiences with working with concrete and visual representations of the addition and subtraction of fractions in isolation that they be engaged in representing everything they do concretely or pictorially in symbolic notation. During this time, it is very important the students, in pairs, small groups and as a whole class, are discussing their ideas, the patterns that they see emerging, and generalizing strategies they can use to determine sums and differences abstractly.  Contexts for problem solving should be varied, and whenever possible the content from other subject areas should be integrated into the mathematics learning to strengthen the student learning in all subjects. | | | |
| **Wh****at Students Should…**  [**Top**](#top) | | | |
| **Know**   * The form for writing a positive fraction is  where a is a whole number and b is a natural number. | | **Understand**   * The denominator (bottom number) of a fraction represents how many equal pieces there are in the whole of the quantity being described by the fraction. * What “finding a common denominator” means. * The relationship between finding common fractions and the identification of factors for the denominators. * Personal strategies for finding sums and differences of fractions. * What it means to write a fraction in “simplified form” and why the new fraction is equivalent to the original. * Why the quantities must be describing the same type of situation when adding or subtracting two quantities. | **Be Able to Do**   * Determine possible common denominators for fractions to be added or subtracted. * Add and subtract common fractions. * Add and subtract fractions and mixed numbers. * Represent solutions to sums and differences of fractions and mixed numbers in different symbolic forms. * Write fractions, and sums and differences of fractions in simplified form. * Determine and explain an estimate for a sum or difference of fractions. |

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| **Key Questions**  [**Top**](#top) |
| * How can two different fractions represent the same quantity? * What does it mean to add/subtract? * What has to be true in order to be able to add/subtract two quantities? * What part of a fraction plays the same type of role as place value in whole numbers and decimals? * How could you estimate the sum or difference of two fractions? * How many different ways can you decompose or compose a mixed number to give different representations for the same quantity? * How could you write a given fraction as a sum or difference of fractions? * How can you determine a common denominator, and when might it be beneficial to do so? |
| **Suggestions for Assessment:**  [**Top**](#top) |
| **Big Idea:**  Adding and subtracting positive fractions and mixed numbers.  **Suggestions for assessment tasks:**   1. Give the students a set of questions that “were completed by a different grade 7 class”. Include correct work and errors (e.g., incorrect common denominators, no common denominators, adding of denominators, and subtracting in the wrong order on parts of mixed number differences). Ask the students to individually correct the questions, using the [*Adding and Subtracting Positive Fractions and Mixed Numbers* *Rubric*](file:///C:\Users\kw426\AppData\Local\Temp\Adding%20and%20Subtracting%20Positive%20Fractions%20and%20Mixed%20Numbers%20Rubric.doc)(or a class developed rubric) to assess the questions. As part of their assessment, ask the students to correct any errors that they find. 2. Have each student create problems involving the addition and subtraction of positive fractions and mixed numbers (at least one problem for each) based upon something they do or have encountered in their lives. The students will need to be given this part of the task ahead of time so that they can think about and research for their problems. Have the students solve the problems using concrete or pictorial models and then explain how those models represent the same strategies that they use when finding sums and differences of fractions symbolically. 3. If the students do the final activity in the **Suggestions for instructional activities** section, their instruction manual could be assessed.   **What to look for:**   * See [*Adding and Subtracting Positive Fractions and Mixed Numbers* *Rubric*](file:///C:\Users\kw426\AppData\Local\Temp\Adding%20and%20Subtracting%20Positive%20Fractions%20and%20Mixed%20Numbers%20Rubric.doc) |
| **Suggestions for Instruction:**  [**Top**](#top) |
| **Big Idea:**  Adding and subtracting positive fractions and mixed numbers.  **Suggestions for instructional activities**   1. Give the students two positive fractions with the same denominators and ask them to represent them concretely or pictorially using any manipulatives or diagrams they feel are appropriate. Have the students walk around the room viewing the different representations and materials used. Encourage the students to ask questions about strategies and reasoning used in different models. 2. Give the students the option of continuing to work with their particular model of the two fractions or to create a new model. As the students explore the adding and subtracting of positive fractions and mixed numbers they should always be allowed to change how they are representing the fractions and the process depending on what they feel will work best for them. As an example, some students will find fraction circles convenient for a cookie question, but coloured tiles more convenient for an area question. Ask the students to use their model to add the two fractions. Before they begin the task, have the class discuss what it means to add and what they think that means that they will need to do with their models. Have at least one student demonstrate each type of model within the class and explain their reasoning. 3. Give the students a new pair of fractions to add, this time with different denominators. Before the students begin, ask them to compare the new fractions to the previous pair. When they notice the difference in the way the denominators are related, ask the students to discuss what the denominator tells them about a fraction. It is very important that the students are thinking about denominators defining the number of equivalent parts as they move into adding fractions with unlike denominators so that they can make sense of trying to find a common denominator. 4. Have the students represent the sum concretely or pictorially and then determine the solution. Have them discuss with a partner the strategies and reasoning that they used. Also have them discuss and show how to record what they did concretely or pictorially in symbolic form. 5. Repeat 3 and 4 using a variety of fractions, with and without common denominators. Have the students share their work and strategies with different partners and then debrief their work with the entire class. During the debrief, tell the students that people often say that to add fractions you need to have common denominators. Ask the students what they think common denominators are and whether or not they agree with the statement and why. Allow the class to discuss, conjecture and debate this topic until the students have a strong understanding of the term and its importance in the adding of fractions. Encourage the students to provide examples during the discussion to reinforce their ideas. 6. Have the students write a journal entry about common denominators and the adding of fractions. 7. Next, starting with the very first pair of fractions, tell the students that you want to subtract the two fractions, or find their difference. Ask the students to write an expression that shows subtraction of the two fractions. Have the students share what they wrote. If all of the students have written the subtraction expression in the order that will produce a positive result (the only kind they can do in grade 7), ask them why no one wrote the opposite direction. If their reason is because they never thought about it, ask them if it would matter if the order were reversed. Have them reflect back on their subtraction of decimals (and whole numbers prior to integers) and ask them if the order would have mattered. If they did not write the other one because they knew it couldn’t be done, have them explain why. In either case, the students should be moving to a concrete or pictorial representation of the two possible expressions to justify which expression they can evaluate right now and which they can’t. Some students might suggest that it is possible to have negative answers. Encourage this thinking by saying that is actually true, but for now we only want to look at evaluating expressions with a positive value. Although it is in grade 9 that students look at all four operations on the rational numbers (positive and negative fractions), it is important that the message that negative fractions don’t exist isn’t given in grade 7. 8. Have the students explore the subtraction of positive fractions concretely and pictorially and then moving into representing the process symbolically using the same sets of fractions as used for the addition of positive fractions. Periodically have the class discuss the strategies and patterns that they are encountering. Provide opportunities for the students to compare the subtraction of positive fractions to the subtraction of whole numbers and decimals as well as to the addition of positive fractions. 9. Have the students write in their journals about their understandings of subtracting positive fractions, including the purpose of common denominators and the significance of the number order in the calculation expression. 10. Give the student a series of mixed numbers and improper fractions and ask them to determine which ones are equivalent. This list of mixed numbers should have two or three sets of equivalent mixed numbers in which different decompositions and compositions of the numbers are present. (e.g., if is included, so might be , , and ). Encourage the students to use any strategies and types of representations (concrete, pictorial, or symbolic) to help them sort the numbers into groups of equivalent fractions. This is a very crucial that students are flexible in their thinking about mixed numbers and improper fractions. They should be able to recognize alternate forms of the same number, and they should be able to recognize different decompositions or compositions just as they have learned in whole numbers. 11. Give the students two mixed numbers, with the same denominator and whose sum will not require any regrouping at the end (e.g., the solution wouldn’t be something like  , but rather would directly work out to , and ask them to find the sum of the two mixed numbers using concrete or pictorial representations. The students may choose to also use symbolic notation, provided they can explain the reasoning behind their work. 12. Have the students discuss the patterns they noticed, and the things that they needed to do or did to find the sum. Encourage the students to focus on what happened to the whole number portions and what happened to the fractional portions when they added the two fractions. If some students changed the mixed numbers to improper fractions have them discuss their reasons for doing so and what strategies and patterns they came up with. 13. Repeat the previous two activities three or four more times (may need more or less experience depending on the class) with new mixed numbers but the same restrictions, this time having the students moving more into the symbolic notation, solving the final one symbolically first and then checking their work by using a concrete or pictorial model. Encourage the students to also try finding the sums using the equivalent improper fractions. 14. Give the students a pair of mixed numbers to add where the sum will give an improper fraction within a mixed number. (e.g., ). Have the students solve the sum using whatever strategies they wish and have them share their answers. Some students may automatically see that the answer is not in proper mixed number form and decompose the fraction part and combine the resulting whole number with the whole number part (e.g.,  because ). Some students will recognize the problem and not know how to solve it, and others will not even notice that there is a problem (a sign that they are just following “the steps” and not thinking about what they are trying to do). Be sure to have the students discuss and explore why such an answer needs to be simplified and how to simplify it. 15. Have the students solve the same question by converting to improper fractions first. Have them compare their results from the two approaches. Tell the students that the convention in mathematics is to write the answer using the same form that the question started (that is, if you started with mixed numbers you end in mixed numbers). Ask the students why this might be. 16. Give the students two mixed numbers that have different denominators. Ask the students to reflect on how this pair of mixed numbers is different from the ones they have been working with and have them hypothesize about how this might impact their addition of the two mixed numbers. Encourage the students to use their experiences with adding positive fractions to predict what will happen in this case. Have the students verify their hypotheses by exploring the addition of the two mixed numbers concretely and pictorially and then representing their work symbolically. 17. Give the students three or four more sets of mixed numbers with different denominators to add. Have the students compare their strategies. If a student or the whole class is always using the same strategy, encourage them to explore alternate strategies as well. This may include encouraging the students to work more in the abstract, or to try using improper fractions (or sticking with the mixed numbers). Along with the students learning to add this type of numbers, this is also an important opportunity to continue the students’ understanding of numbers in general. 18. Give the students two mixed numbers with the same denominators and the subtraction in mixed number form will not require decomposing one of the numbers and ask them to write a subtraction statement that they could evaluate (e.g.,  and ). Have the students discuss what they have written, having them reflect back on their conversations about the subtraction of positive fractions. Again, if a student argues that it is possible to have a negative mixed number, do not disagree, but bring them back to the topic of focus by asking them the more specific question of how to write the subtraction statement so that the result is positive. 19. Have the students work in pairs to discuss their strategies and models for representing the sum of the two mixed numbers. Encourage the students to record the processes they use symbolically. Also encourage the students to consider finding the differences using improper fractions (or mixed numbers if they are always using improper fractions). Have the class discuss and share their strategies and learnings after they have completed the questions. 20. Give the students a subtraction statement involving two mixed numbers with the same denominators such that the subtraction in mixed number form will require the decomposition of one number (e.g., and ). Have the students try to find the sum symbolically. If they subtract the numerators as they are and just write the positive of the result (e.g., gives a numerator of 4 when the difference is -4), have the students represent the subtraction of the fractional parts only (in the order that they are in the expression given) and have them discuss whether that is possible. Some students may be able to solve it symbolically and in those cases ask the students what they did and what they were thinking about when they did it. Get those students to justify their reasoning using a concrete or pictorial model. If it hasn’t emerged in the previous discussions, remind the students of the sorting activity done earlier and ask them if there is a strategy they could use from that activity to solve the problem they’ve encountered. Let the students play with the question using concrete and pictorial models. Have the students share the strategies that they develop. 21. Give the students two or three more questions like the previous one to solve and have them share their work and reasoning with a partner. Encourage the students to explore alternate strategies such as writing the mixed numbers as improper fractions. 22. Pose the question: What if the mixed numbers didn’t have the same denominator? Have the students discuss and strategize what they would do. Encourage the students to create questions that involve mixed numbers with different denominators and then explore the ideas that they have come up with. Have the students share their strategies and reasonings. The students can exchange their questions with each other and compare and discuss their strategies for finding the answer. 23. Have the students write their own “How to Add and Subtract Positive Fractions and Mixed Numbers” instruction manual. Tell the students that before starting to write the manual, they should outline what they think are the important understandings that someone learning how to add and subtract fractions needs to know and think about. Remind them to include examples, and to show alternative examples. |