**T****he Number Strand: Outcome N7.3**

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| **Outcome** | **Indicators** |
| N7.3 Demonstrate an understanding of the relationships between positive decimals, positive fractions (including mixed numbers, proper fractions and improper fractions), and whole numbers.  [C, CN, ME, R, T]  *In support of the K-12 Mathematics goals of Number Sense and Logical Thinking and Mathematical Attitude.* | 1. Predict the decimal representation of a fraction based upon patterns and justify the reasoning (e.g., knowing the decimal equivalent of 1/8 and 2/8, predict and verify the decimal representation of 7/8). 2. Match a set of fractions to their decimal representations. 3. Sort a set of fractions as repeating or terminating decimals. 4. Express a fraction as a terminating or repeating decimal. 5. Express a repeating decimal as a fraction. 6. Express a terminating decimal as a fraction. 7. Explain and demonstrate how any terminating decimal can also be written as a repeating decimal and explain how this impacts their understanding of writing decimals for fractions. 8. Explain why fractions and decimals are related through division. 9. Provide an example where the decimal representation of a fraction is an approximation of its exact value. 10. Order a set of numbers containing decimals, fractions, and/or whole numbers in ascending or descending orders and justify the order determined. 11. Identify with justification a number that would be between two given numbers (decimal, fraction and/or whole numbers) in an ordered sequence or shown on a number line. 12. Identify incorrectly placed numbers within an ordered sequence or on a number line. 13. Order the numbers in a set of numbers by using benchmarks on a number line such as 0, and 1. |
| **Learning Space** [**Top**](#top) | |
| In grade 5, the students studied the relationship between fractional and decimal forms of numbers to thousandths. In grade 6, the students explored the relationship between mixed numbers and improper fractions. The intent of this outcome is to create and consolidate the students’ understandings of the relationships in grades 5 and 6. This outcome also has a direct tie to another grade seven outcome N7.4 “Expand and demonstrate an understanding of percents to include fractional percents between 1% and 100%” in which students will be considering the relationships between percent and its possible representations as a fraction or decimal.  It is important that the students learning with respect to this outcome be developed from a concrete to abstract level. This means that students need to make, test, and verify conjectures regarding how the different types of numbers can be related and ordered. The students should begin by exploring the relationships between fractions, decimals, and whole numbers using concrete and pictorial representations which can then be described symbolically leading to the students creating personal strategies for relating numbers from the different sets.  A wide range of contexts should be used in exploring the relationships between decimals, fractions and whole numbers, many of which can emerge through the students’ studies in other subject areas. Measurement contexts can provide a very rich environment in which students can explore and relate the different number types while also developing an understanding of the concept of the number line and every number having a specific “home” on that line. | |

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| **What Students Should…** [**Top**](#top) | | |
| **Know**   * Identify a whole number in simplified form (i.e., not given as a decimal or fraction). * Identify a quantity written in fractional form. * Identify a quantity written in decimal form. * Identify a quantity written as a mixed number. * Identify a quantity written as an improper fraction. | **Understand**   * How fractions and decimals are different ways to symbolically represent quantities that represents a combination of parts of wholes and wholes. * Why whole numbers are fractions with specific characteristics. * Why improper fractions and mixed numbers are two different symbolic ways to represent a particular type of quantity (one that combines wholes with non-zero parts). * Between any two non-equal fractions, non-equal decimals, or a fraction and decimal which are not equal, there are many fractions and decimals whose quantity is larger than one of the other quantities, but smaller than the second quantity. * Some fractions represent a decimal whose exact value cannot be written out but can be indicated (using symbolic devices such as the line over top of the repeating decimals) * How a terminating decimal is has an equivalent repeating decimal**.** * All fractions can be written as a decimal (terminating or repeating OR repeating only – depending on how you chose to define the decimals). | **Be Able to Do**   * Change between different types of representations for decimal numbers and fractions (i.e., write any type of fraction as a decimal and any type of decimal as a fraction). * Order a set of fractions and decimals in increasing or decreasing order and explain the reasoning used. * Locate a set of fractions and decimals on a number line using given or self-selected benchmarks on the number line (such as 0, 1, 5, 10). * Determine at least one fraction or decimal between two non-equal fractions, two non-equal decimals, or one fraction and one decimal that are not equal. |
| **Key Questions** [**Top**](#top) | | |
| * How are decimals and fractions related to each other? * How are improper fractions and mixed numbers related to each other? * How are improper fractions and mixed numbers different from other fractions? * What is the relationship between terminating and repeating decimals? * How do you know where to place a given number (fraction, decimal, or whole number) on a number line? * What is a number line? * Which is more exact, a fraction or decimal? Explain your reasoning. * How is the word “and” in the naming of a decimal number related to the word “and” in the meaning of a mixed number? | | |
| **Suggestions for Assessment:** [**Top**](#top) | | |
| **Big Idea:**  Positive rational numbers.  **Suggestions for assessment tasks:**   1. As a homework or research task, have the students look for examples in their homes and in the media of at least two of each of whole numbers, fractions, decimals, mixed numbers, and improper fractions. Have the students write about the context of each of the numbers and give their hypothesis about why that particular format was chosen. 2. Have the students order the set of numbers they collected in their homework or research. 3. Have the students select one of each of their examples for the five types of numbers that they collected and ask them to write that number in an alternate symbolic form, making sure that in the end they have all five forms represented. 4. Ask the students to critique the statement “all fractions can be written as repeating decimals.” In their arguments for or against the truth of this statement, students should be expected to provide examples and explain their reasoning.   **What to look for:**   * See [*Positive Rational Numbers Rubric*](file:///C:\Users\kw426\AppData\Local\Temp\Positive%20Rational%20Numbers%20Rubric.doc)*.* | | |

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| **Suggestions for instruction:** [**Top**](#top) |
| **Big Idea:**  Positive rational numbers.  **Suggestions for instructional activities**   1. Provide the students with a fraction and ask them to write it in decimal form. Be sure to have manipulatives and materials available for the students to create pictures and other models if they desire. Have the students discuss their strategies. Have the students repeat the task with a few more fractions, making sure that both terminating and repeating decimals are represented. Also include one or more fractions whose decimals will require the students to go beyond thousandths. This will be an extension of the students’ grade 5 learnings which went only to hundredths. While debriefing their work, ask the students to reflect on how they could make a terminating decimal a repeating decimal and whether it is possible to do the reverse. 2. Provide the students with a decimal and ask them to write it in fractional form. Be sure to have manipulatives and materials available for the students to create pictures and other models if they desire. Have the students discuss their strategies. Have the students repeat the task with a few more decimals, being sure to include repeating and terminating decimals. Also include at least one decimal that goes beyond thousandths. This will be an extension of the students’ grade 5 learnings. Debrief the students’ work to ensure that they have strategies that they are able to use. 3. Pose a question such as: “which is greater 0.33 or?”. Have the students work on the question on their own, then in pairs, and finally discuss it as a class. If none of the students have used a concrete or pictorial representation to explore the question, ask them to try to develop models to represent and compare the values to confirm their reasoning. Repeat the same task with different decimals and combinations of fractions and decimals. Debrief by having the class create a list of strategies and things to consider in comparing repeating and terminating decimals as well as fractional decimals. 4. Provide the students with a mixed number and ask them to write down in words how that number is read (e.g.,  is read “five and twenty-seven hundredths”). Next give the students the equivalent decimal and ask them to write down in words how it is read (5.27 is read five and twenty-seven hundredths). Have the students compare the names of the numbers and ask them which of the two numbers is greater and how they know. It is important that the students are saying the word “and” to indicate the break between the whole number and the fractional portion in the mixed number as well as to indicate the decimal point separating the whole number and decimal portion of a decimal. It is also important that they recognize which part of the mixed number contributes to the whole number portion of the decimal and which part of the mixed number contributes to the decimal portion of the decimal. |

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| 1. Give the students a list of mixed numbers and a list of decimal equivalents and have them match the mixed numbers to their equivalent decimals. Make some of the matches obvious (that is where the fractional part of the mixed number has a denominator that is a multiple of 10), somewhat obvious (the fractional part of the mixed number can be changed to an equivalent fraction that has a denominator that is a multiple of 10) and much less obvious (the fractional part cannot be written with a denominator that is a multiple of 10). Overall, using the obvious fractional parts and the whole numbers, the list should be such that the students can reason which decimals and mixed numbers match. 2. Next, tell the students that you are going to want them to prove their matches are correct without referring to the whole number portions of the mixed numbers and decimals. Ask the students what they must prove (that the decimal portions are equal to the fractional portion). Have the students work in pairs to justify their choices based on the decimals and fractions. Have the class come back together to discuss their strategies and reasoning. 3. Give the students an improper fraction and tell them that they are to write it as a decimal. Ask the students to try to find as many strategies as possible for changing from improper fraction form to decimal form. After students have worked on the task individually and in pairs, have the entire class discuss their strategies and ideas. At the least, each student should see that division (using technology) is one method and that another method is to write the improper fraction as a mixed fraction first and then change the mixed fraction into a decimal. 4. As a homework assignment, give the students a whole number and tell them that their task is to write an equivalent number for that whole number in: decimal form, fractional form, improper fractional form, and mixed number form. Have the students share and justify their results. Let the class consider alternate possibilities. (e.g., 5 could be, , …) . Developing flexibility in the decomposing of rational numbers can be of great help in students’ development of understanding of operations on rational numbers. 5. Provide the students with a list of rational numbers in a variety of forms. Ask the students to order the rational numbers by placing them on a number line. Have the students work on the task on their own, and then discuss it as a whole class by constructing the number line on the board and placing the numbers on it. Have students suggest placements for each of the numbers and to provide their reasoning for their decision. Encourage the students to challenge the placement of numbers and to also provide alternate strategies they used. In pairs, have the students generate their own list of rational numbers to be sorted and then have each student sort the numbers and put them on the number line. 6. Have the students write in their journal about what the position of a number on a number line tells them about the meaning of the number. |

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| 1. As a homework assignment, give the students two rational numbers and ask them to do two things with those numbers:  * record the numbers in increasing order * determine a number between the two numbers and represent it in decimal, fractional form (note: the fractional form may be a common fraction, improper fraction, or mixed number depending on the numbers that you provide).   Discuss their findings and strategies at the beginning of the next class. Provide the students with additional pairs of numbers to repeat the task with, or have the students go in pairs with each student creating a challenge problem for each other. |