**The Number Strand: Outcome N7.2**

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| **Outcome** | **Indicators** | | |
| N7.2 Expand and demonstrate understanding of the addition, subtraction, multiplication and division of decimals to greater numbers of decimal places, and the order of operations.  [C, CN, ME, PS, R, T]  *In support of the K-12 Mathematics goals of Number Sense and Logical Thinking and Mathematical Attitude.* | 1. Provide a justification for the placement of a decimal in a sum or difference of decimals up to thousandths (e.g., for 4.5 + 0.73 + 256.458, think 4 + 256, so the sum is greater than 260, thus the decimal will be placed so that the sum is in the hundreds). 2. Provide a justification for the placement of a decimal in a product (e.g., for $12.33 × 2.4, think $12 × 2, so the product is greater than $24. Thus, the decimal in the final product would be placed so that the answer is in the tens). 3. Provide a justification for the placement of a decimal in a quotient (e.g., for 51.50 m ÷ 2.1, think 50 m ÷ 2, so the quotient is approximately 25 m. Thus, the final answer will be in the tens). *(Note: if the divisor has more than one digit students should be allowed to use technology to determine the final answer.)* 4. Solve a problem involving the addition or subtraction of two or more decimals. 5. Solve a problem involving the multiplication or division of decimals with 2- digit multipliers or 1-digit divisors (whole numbers or decimals) without the use of technology. 6. Solve a problem involving the multiplication or division of decimals with more than a 2-digit multiplier or 1-digit divisor (whole number or decimal), with the use of technology. 7. Check the reasonableness of solutions using estimation. 8. Solve a problem that involves operations on decimals (limited to thousandths) taking into consideration the order of operations. 9. Explain, by using examples, why it is important to follow a specific order of operations when calculating with decimals and/or whole numbers. | | |
| **Learning Space** [**Top**](#top) | | | |
| Students will have begun learning about decimals and relating them to quantity in grade 4, moving into addition and subtraction of decimals in grade 5 and multiplication and division by one-digit natural numbers in grade 6. In grade 7, the students are expanding their understandings from the past three grades by moving to multiplication and division by decimals and by solving computations and problems that involve more than one operation on decimals.  Having already encountered the concept of “order of operations” with respect to whole numbers in grade 6, the students should be encouraged to use this prior knowledge to extend their understanding of the order of operations to expressions involving decimals. Students should be generalizing their own strategies and rules that they can efficiently and effectively use to carry out multiple computations in a single expression based upon exploration and past experiences. The students should be encouraged to view the order of operations involving decimals as being consistent with the order of operations for whole numbers.  The other major new component to this outcome for the students is the notion of multiplication and division by decimals. Students need to be engaged in experiences that will help them relate multiplication and division by decimals to equivalent statements of multiplication and division with whole numbers. In exploring this idea, students should be encouraged to make direct connections to their prior learnings about the preservation of equality in grade 6 as well as those in grade 7. Concrete and visual representations, along with discussions highlighting student thinking and reasoning should be core to this learning.  This outcome also provides a great deal of opportunity for students to expand and refine their mental mathematics skills. Emphasis should be put on understanding the placement of the decimal point in solutions to decimal computations based on estimation and not upon a rule that the students have been given. This will mean the use of high level questions that require the students to reflect upon and use prior knowledge.  Decimal values should be included in the students’ study of circumference, areas, linear patterns and measures of central tendencies to provide the students with additional meaningful contexts for understanding the four operations and their order when dealing with decimal values. Similarly, connections can and should be made between mathematics and other subject areas as situations arise in which decimals are being used. Measurement of lengths and time are often recorded in decimals and can provide interesting contexts for problems for the students to solve. | | | |
| **What Students Should…** [**Top**](#top) | | | |
| **Know**   * The order of operations is identical whether dealing with whole numbers, decimals, or both. | | **Understand**   * Why it is necessary to have an order of operations for carrying out calculations with decimals. * The relationship between the placement of the decimal and the whole number approximation of a calculation involving decimals. * The strategies for dividing by more than a single digit divisor is the same regardless in intent as division by a single digit. * Why they are allowed to use technology for division by more than one digit numbers. * What type of operation (or types of operations) are involved in a particular context or situation. * Why dividing by a decimal (e.g., 0.2) results in a larger value. | **Be Able to Do**   * Carry out calculations involving decimals (up to thousandths) involving multiple operations and groupings of operations * Check and justify the reasonableness of calculations involving decimals * Use operations on decimals to find solutions to questions and problems * Generalize and verify strategies for carrying out operations on decimals. * Explain the strategies used to perform calculations involving decimals * Explain strategies and reasoning used when multiplying and dividing by a decimal number. |
| **Key Questions** [**Top**](#top) | | | |
| * Why is there a set of order of operations to be done when carrying out multiple computations in an expression? * How are multiplication and division related to addition and subtraction and how does that relationship impact the order of operations? * What are the similarities when doing computations with decimals to doing computations with whole numbers? * How can you determine the placement of the decimal point in computations involving decimals? * Do you agree with the statement that division always results in a smaller number and multiplication always results in a larger number? What cases do you need to consider when answering this question? * Why might you chose to use a calculator to do some calculations and what are the benefits and disadvantages of using technology? | | | |
| **Suggestions for Assessment** [**Top**](#top) | | | |
| **Big Idea:**  Determining sums, differences, products and quotients of decimals.  **Suggestions for assessment tasks:**   1. Provide the students with rulers or metre sticks and ask them to collect measurements from around the classroom. Using the measurements that they collect, have the students write and solve problems that involve the four operations on decimals. The problems may include more than one operation, but over all of the problems each of the operations must be included. If the students choose to use a calculator, have them explain their reason for the choice. They should be expected to carry out at least three of their computations without the use of a calculator. 2. Have the students record a journal entry describing how they know where the decimal point belongs in answers involving each of the four operations on decimals. The students may wish to include examples and use concrete or pictorial models to help explain the reasoning. 3. Provide the students with a decimal computation involving more than one operation and requires the students to follow the order of operations. Have the students carry out the computation. Then, provide the students with a different solution. If the student carried out their computation using the order of operations give them a solution where the order of operations wasn’t used. If the students did not follow the order of operations in doing their computation give them a solution where the order of operations were followed. Have the students compare their solution to the solution you gave them and ask them to explain the differences between the solutions and decide which of the two is correct and explain why.   **What to look for:**   * See [*Determining Sums, Differences, Products, And Quotients Of Decimals Rubric*](file:///C:\Users\kw426\AppData\Local\Temp\Determining%20Sums%20Differences%20Prods%20Quots%20Of%20Decimals%20Rubric.doc)*.*   **Big Idea:**  Estimating sums, differences, products and quotients of decimals.  **Suggestions for assessment tasks:**   1. Have a set of four problems that involve an estimate of one of the operations involving decimals. Randomly assign one problem to each student. Have the student solve the problem. This problem should be assessed for the student’s recognition that the context requires estimation and their ability to use estimation since assessment tasks from the previous big idea consider their ability to carry out computations involving the four operations. 2. Provide the students with a computation question and a statement of the computation that a student was going to use to estimate the value (e.g., 0.23 x 0.8 and 0.2 x 0.8) and ask the students whether the result of the estimated computation will be greater or smaller than the actual computation and why.   **What to look for:**   * See [*Estimating Sums, Differences, Products, And Quotients Of Decimals Rubric*](file:///C:\Users\kw426\AppData\Local\Temp\Estimating%20Sums%20Differences%20Products%20Quotients%20of%20Decimals%20Rubric.doc)*.* | | | |

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| **Suggestions for Instruction** [**Top**](#top) |
| **Big Idea:**  Determining sums, differences, products and quotients of decimals.  **Suggestions for instructional activities**   1. From their previous study of decimals including addition and subtraction of decimals, the students should be familiar with different concrete and pictorial models for decimals. If this is not the case, it is necessary to have them explore these models as ways to represent decimals before moving into exploring multiplication and division of decimals. 2. Multiplication and division of decimals is new to grade seven students, so it is important that they experience modelling these operations concretely and pictorially and develop their own strategies for carrying out these computations symbolically. It is very important that the modelling of these two operations on decimals be directly tied to the ideas of modelling multiplication and division on whole numbers. The students could start by considering an example of whole number multiplication (e.g., 12 x 8). Have the students recall the strategies that they used to explore and carry out this type of product. Concretely or pictorially, the students should be familiar with making an area or array using base ten blocks or their pictures to form the dimensions of 12 (1 rod and 2 units) and 8 (8 units). Also have the students reflect on how they represent what is shown on the concrete or pictorial models using symbolic notation and on the development of the distributive property for multiplication over addition. Next, ask the students to try (in pairs) a multiplication involving a decimal (e.g., 12 x 0.8) by modelling it either concretely or pictorially. Have the students compare the results that they get and discuss the impact of the addition of the decimal place. Have the students also consider whether the distributive property still applies and if so what information it reveals (e.g., 12 x 0.8  (10 + 2) x 0.8 10 x 0.8 + 2 x 0.8  8 + 1.6 Some things to note: multiplying the tenth by ten results in ones, and 2 x 0.8 is the same as 2 x 8 except for the decimal place). Have the pairs discuss their ideas with another pair of students. Provide another multiplication question for the students to explore, encouraging them to do four things for each question:  * model it concretely or pictorially * compare it to multiplying whole numbers with the same digits * consider what the distributive property reveals * consider the relationship between the original question and the placement of the decimal point.   Have the whole class discuss their ideas and discoveries.   1. Give the students more questions to consider, or using dice or cards the students can generate their own questions. Gradually add a decimal place into the second multiplicand and later increase the number of decimal places in each of the multiplicands. The students may either continue to work in pairs, work individually and share their ideas with another student, or work in groups of three or four. Ask the students to keep a journal of the ideas, conclusions and questions that they have regarding multiplication involving decimals. 2. As a class, have the students share and discuss the information that they recorded in their journals. As individuals and as a class, have the students generate strategies for determining the products of decimals symbolically by referring back to the properties they discovered in the concrete and pictorial models. 3. Repeat these first three instructional activities with division by 1 digit divisors as the focus. Some students may be able to make predictions about the placement of the decimal prior to their actually modelling the divisions. Invite them to do so with the intent being that they try to verify their predictions. 4. Give the students a whole number computation that involves a number of operations in which the order of operations comes in to play. Have the students carry out the computation individually and then compare their solutions. Have the students resolve conflicts in the solutions. Then ask them to carry out the same computation, but change some of the numbers involved into decimals, keeping in mind the point is the order of operations and not the actual individual computations. Have the students carry out the computation and share their results with a partner. Then as a large class discuss their results and the extension of order of operations to all types of numbers. 5. In groups of 4 to 6, have each student develop a computation that involves more than two operations, decimals (whole numbers can also be included), and requiring the order of operations to be considered as well as a solution to their question. Have each student record all of their groups’ questions and carry out each of the computations. Each student then becomes responsible for assessing the solution to their question by each of their group members. The group is responsible for coming to consensus regarding the correct solution for each question. 6. Have each student create a set of problems involving computations with decimals based on situations that they are familiar with. Have the students share and solve their questions with a partner.   **Big Idea:**  Estimating sums, differences, products and quotients of decimals.  **Suggestions for instructional activities**   1. Give the students a division statement that involves division by a decimal with 2 digits, and ask them if they think the answer given is correct or not (your choice if it is). Have the students discuss how they could verify the answer without actually doing the computation. In this discussion, the students may need to be reminded about their discoveries regarding the placement of the decimal point in computations involving decimals. They may also need to have the suggestion to consider an easier question first which would be the result of rewriting the original statement in whole numbers. As the students begin to talk about estimating the value of the numbers involved in the statement to check the reasonableness of the answer, give them other statements to check involving the different operations and combinations of operations. Have the students discuss their strategies for determining the estimates. Different estimations can be used for the same numbers, but some will be better to choose because of the answer. Have the students discuss why they chose the particular estimates. 2. Return the students to the original division statement by decimals with 2-digits. Although the students are not to be assessed on dividing by 2-digits for this outcome, it is important that they develop an understanding of how division by more digits develops as an extension of division by 1-digit. Have the students explore their ideas of how they could reach the solution to the question using manipulatives, pictures, or symbols. Encourage the students to discuss their reasoning and ideas with a partner or in a small group. Have the students try their strategies on a new division question that doesn’t have the decimal given. Ask the students to verify the reasonableness of their answers. Then, ask the students how they could check that the solution is correct and not just reasonable. Have the students use their calculators to check their answer. Have a discussion as a class about when it’s appropriate and when it’s not appropriate to use calculators and other technology. Tell the students that they should be able to verify the reasonableness of division by 2-digits, but they will be allowed to use calculators to determine the actual value of the computation. 3. Give the students a problem such as “A shopper wanted to know about how much it would cost to buy 4 avocados costing $0.97 each.” and ask them to give their answer. Most of them, because of the familiarity of the type of situation will likely say $4 quite easily. Have the students explain their reasoning for the solution. Next show them what the shopper did (worked out 4 x 0.97 and then rounded up). Ask the students to discuss which solution strategy is the best way to get the answer according to the intent of the shopper’s question in the situation. 4. Next, have the students brainstorm places where they see decimals all the time (money, measurements, time…) and ask them to create two problems, one where an estimated computation is being sought, and one where the actual value of a computation is being sought. Have the students go into pairs and switch problems without telling their partner which problem is which. Have the students discuss their partner’s approach to solving the problems. Debrief by having the students share how they determined when to estimate and when not to, as well as their estimation strategies. 5. Provide the students with a combination of lone computations and problems in which the student must decide whether or not to estimate. Also include computations that require the students to consider the order of operations and problems that include division by 2-digit numbers. |