**The Number Strand: Outcome N7.1**

[Learning Space](#ls) [What Students Should . . .](#wss) [Key Questions](#kq) [Assessment](#assessment) [Instruction](#instruction)

|  |  |
| --- | --- |
| **Outcome** | **Indicators** |
| N7.1 Demonstrate an understanding of division through the development and application of divisibility strategies for 2, 3, 4, 5, 6, 8, 9, and 10, and through an analysis of division involving zero.  [C, CN, ME, R]  *In support of the K-12 Mathematics goals of Number Sense and Logical Thinking and Mathematical Attitude.* | 1. Investigate division by 2, 3, 4, 5, 6, 8, 9, or 10 and generalize strategies for determining divisibility by those numbers. 2. Apply strategies for determining divisibility to sort a set of numbers in Venn or Carroll diagrams. 3. Determine or validate the factors of a number by applying strategies for divisibility. 4. Explain the result of dividing a quantity of zero by a non-zero quantity. 5. Explain (by generalizing patterns, analogies, and mathematical reasoning) why division of non-zero quantities by zero is not defined. |
| **Learning Space** [**Top**](#top) | |
| Prior to grade 7, students have had a great deal of experience exploring and developing an understanding of division as a numeric and symbolic way of demonstrating and determining quantities in contexts involving sharing and grouping. The goal of this outcome is to have the students focus in on the properties of numbers and how they relate to division.  Students need to explore visually, concretely, and symbolically numbers that are and are not divisible by a number (including 2, 3, 4, 5, 6, 8, 9 and 10). Through these explorations, the students should be generalizing patterns that they see to form strategies for determining divisibility by 2, 3, 4, 5, 6, 8, 9 or 10 and applying the strategies to determine factors for whole numbers. The process of generalizing the patterns into strategies needs to emphasize hypothesizing, testing, and reasoning so that the students continue to develop their understanding of mathematical thinking.  By using Venn and Carroll diagrams, students can begin to explore relationships between whole numbers in terms of divisibility and factors.  Division involving zero (as a divisor – what you divide by, and as the dividend – what you are dividing into) is an understanding in mathematics that is foundational in a variety of contexts. Division involving zero should be explored in the same way that division involving any other whole number has been: by modelling (concretely, visually, and symbolically), exploring and generalizing patterns, and developing the ability to support conclusions and answers. The relationship between multiplication and division, as well as the purpose or meaning of division (as grouping or sharing) need to be revisited and used as the basis for developing ideas regarding division involving zero. | |

|  |  |  |
| --- | --- | --- |
| There will be numerous opportunities to bring the students’ understanding of divisibility into other subject areas when they explore problems related to the content from the other subject areas. Similarly, the contexts for problems from other subject areas can be used in the mathematics classroom. The divisibility of numbers is also of importance in many of the other content areas in grade 7 mathematics, including determining and describing patterns, solving one-step single-variable linear equations, creating circle graphs from data sets, determining the area of triangles, and determining the mean for a set of data. Being aware of the students’ learnings about divisibility when studying these content areas provides the teacher with many opportunities within meaningful contexts to reinforce what the students have learned. | | |
| **What Students Should…** [**Top**](#top) | | |
| **Know**   * The term *factor* is the name for any number that divides evenly into given number, e.g., for the number 12 all of the following whole numbers are called factors of 12: 1, 2, 3, 4, 6, and 12 * The terms “Carroll diagram” and “Venn diagram” and be able to identify an example of either one. * The term *undefined* (or *does not exist*). | **Understand**   * How patterns discovered in carrying out division tasks can be generalized into rules for determining divisibility. * The relationships shown within a Carroll diagram. * The relationships shown within a Venn diagram. * The relationships among the divisibility rules, factors, multiplication, and division. * Why division by zero cannot be done but division into zero can be done. * How finding factors is related to determining equal groupings or sharing. | **Be Able to Do**   * Determine the factors of numbers using divisibility rules that they have generalized. * Create and use Carroll diagrams to solve problems involving divisibility. * Create and use Venn diagrams to solve problems involving divisibility. * Provide justification for whether a value is a factor for a given whole number. |
| **Key Questions** [**Top**](#top) | | |
| * How can knowing divisibility rules (or patterns) be useful? * In a Venn diagram illustrating the factors of two different numbers, what is significant about the factors in the area of intersection (overlap)? * What does it mean to divide by a number? * How can you determine if one number is a factor of another? | | |

|  |
| --- |
| **Suggestions for Assessment** [**Top**](#top) |
| **Big Idea:**  Nature of Division  **Suggestions for assessment tasks:**   1. Have the students create a presentation (oral, written, art, dance, song, drama...) to demonstrate their understanding of divisibility and the strategies they use to determine factors for a number. These presentations could be assessed using a class designed rubric, checklist, or rating scale. Criteria used for the assessment of the performance might include: originality, correctness, completeness, and clarity. 2. Ask the students to create a Venn diagram and/or Carol diagram for a given pair of numbers and their factors. 3. Ask the students to identify, with justification, which of a given list of whole numbers are the factors of another whole number. 4. Ask the students to determine the factors common to three or more whole numbers and to come up with a variety of ways to represent their solutions. Have the students explain their strategies for determining the common factors. 5. Have the students explain why division by zero is not defined.   **What to look for:**   * See [*Nature of Division Rubric*](file:///C:\Users\kw426\AppData\Local\Temp\Nature%20of%20Division%20Rubric.doc)*.* |
| **Suggestions for Instruction** [**Top**](#top) |
| **Big Idea:**  Nature of Division  **Suggestions for instructional activities**   1. It is desirable that students discover strategies for determining divisibility rather than being given rules to apply without understanding the origins of the rules. To attain this type of learning, students could be placed in groups that explore divisibility by one, two, three, four, five, six, eight, nine, and ten. First, the students would need to do division, either by hand, with the use of technology, or both, to determine which of a set of whole numbers is divisible by their group’s number. Each group could then be given a list of different patterns, some useful and some not, they can investigate in relation to the numbers they found were divisible by the group’s number. For example, this list would include the standard strategies: is it an even number, if you add up the digits do they give a number divisible by three, and do all the numbers end in the same number, as well as dead-end strategies such as are the digits in the number in increasing or decreasing order? 2. Have the students share and compare their strategies for determining the divisibility of numbers. Keep returning to the question of what division and divisibility really mean. Ask the students to reflect on why divisibility rules might be helpful to know. 3. Give the students a set whole numbers and ask them to find all of the factors for the numbers (without the use of technology). Have the students share their results and strategies. If the student did not use the divisibility rules to decide whether to try dividing by a number, ask the students to hypothesize how the divisibility rules could have been used to help them determine the factors of the numbers. 4. Provide the students with a list of whole numbers and ask them to consider the divisibility of those numbers with respect to 2 and 3 (or any other pair of factors). Ask the students to explore ways to represent their results pictorially (such as in a Venn diagram or Carroll diagram). 5. Have the students work in pairs giving each other whole numbers to test for divisibility and checking each others solutions. 6. To explore division involving zero, begin by having the students rewrite some given division statements not involving zero as multiplication statements and vice versa (e.g., 24  3 = 8 can be written as 3 x 8 = 24). Next, ask the students about the result of dividing into zero (e.g., 0  5 = ?). As the students generate answers, ask them to verify their results using multiplication. When the students have done enough examples, they should be able to generalize that zero divided by any number not equal to zero is zero. The students may want to say just any number as the divisor but this is incorrect because of zero divided by zero being indeterminant. This is an example, which because of the students’ current level of mathematics, must be told to them (Note: they do not need to know the word indeterminant). At this point, it is important to ask the students to provide examples and/or models to make sense of the solution always been zero. 7. Next asked the students what a whole number divided by zero equals (e.g., 80 = ?). Most, if not all students, will likely say that the answer is zero. A few might also give the answer of 8. It is unlikely that students will recognize that there is no real solution to this question. Ask the students to verify their answers using multiplication. They should quickly realize that their answers are incorrect. From this point, suggest to the students that in pairs or small groups they should discuss, explore, and try to represent division by zero. The teacher may need to give the students some thinking prompts such as: how have you modelled division before, what does it mean to divide, and does thinking about multiplication help you? Ask the groups and individual students to share things they discovered, what they tried, and what problems they encountered. Discuss the ideas and strategies shared asking probing questions to clarify and redirect thinking. Together the class should come to the conclusion that there is no answer to divide by zero. Many students will say that the answer is infinity. This is another situation where the student’s current mathematical knowledge does not allow them to understand that it is not infinity because negative numbers give a different result. It is important then that the students be told that division by zero is “undefined” or “does not exist”. |