**The Patterns and Relations Strand: Outcome P4.2**

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| **Outcome** | **Indicators** | | |
| P4.2 Demonstrate an understanding of equations involving symbols to represent an unknown value by:   * writing an equation to represent a problem * solving one step equations.   [C, ME, PS, R]  *In support of the K-12 Mathematics goals of Number Sense, Logical Thinking and Mathematical Attitude.* | 1. Explain the purpose of the symbol, such as a triangle or circle, in an addition, subtraction, multiplication or division equation with one unknown (e.g. 36 ÷ = 6). 2. Write an equation in symbolic form for a given pictorial or concrete representation. 3. Identify the unknown in a story problem, represent the problem with an equation, and solve the problem concretely, pictorially, or symbolically. 4. Create a problem in context for an equation with one unknown. 5. Solve a one-step equation using manipulatives. 6. Solve a one-step equation using guess and test. 7. Explain what is meant by “one-step equation with one unknown”. 8. Represent and solve an addition or subtraction problem involving a “part-part-whole” or comparison context using a symbol to represent the unknown. 9. Represent and solve a multiplication or division problem involving equal grouping or partitioning (equal sharing) using a symbol to represent the unknown. | | |
| **Learning Space:** [**Top**](#top) | | | |
| In grades 1 and 2, students have explored the notion of equality and inequality. This is a very important understanding for the students to have, and as they move into the writing and solving of equations it is important for the students to reflect on what the equality sign in the equations means. Students need to understand that the equal sign is not used to convey the meaning “do the work,” but that it guarantees a balance or equality between the representations on the two sides of the equation. If students are misusing the equal sign, or misinterpreting it, invite them to develop their own notation for indicating that they are “doing the work”, such as an arrow pointing to the resulting step. The teacher can also help the students to maintain a correct understanding of the equal sign by choosing their wording carefully so that they are communicating that the equality must be maintained.  In grade 3, the students began solving equations that involved either addition or subtraction. In grade 4, their experiences are to be extended to solving equations using any one of the four operations, and to write and solve equations related to word problems. At both these grades, a shape not associated with numbers or letters should represent the unknown value. For example, a square, triangle, or heart might be used. Teachers should also be careful to not use symbols that represent mathematical ideas, such as the sign for perpendicular. Students should be regularly engaged in communication opportunities (oral or written) in which they explain the role of the symbol for the unknown value. Students should see a wide variety of symbols being used, and should be encouraged to think of new symbols that they might use.  After learning the meaning of the numbers, the solving of equations is one of the next most abstract mathematical concepts that the students learn. It is very important therefore that the students be engaged in learning experiences that require a wide variety of representations. Questions such as “how might you show me what is happening in this problem?” become very important for the students to develop abstract algebraic representations. As well, students should be engaged in creating and identifying alternate representations. Finally, students need to understand the connection between the concrete, physical, and pictorial represents that they create and the mathematical symbolic representation of an equation. This requires students to have experiences making connections between the different forms of representation and communicating their reasoning in doing so.  The problems that students solve should come from a variety of situations that are meaningful to them. These problems might arise from personal experiences, mathematical quandaries (“I wonder what would happen if…,” or “How could you represent what is shown on the two sides of this balance?”), and contexts related to the students’ studies and discussions in other subject areas. As well, the problems the students solve should not just be teacher initiated, but created by students as well. | | | |
| **What Students Should…** [**Top**](#top) | | | |
| **Know**   * symbols, such as squares, triangles, and circles in equations represent an unknown quantity. * an equal sign implies equality, not “do the question”. | | **Understand**   * the quantities represented by both sides of an equation are the same amount (hence the equal sign). * if a value is substituted into the role of the symbol in an equation and the two sides of the equation remain equal (the same amount), then the value is a solution to the equation (one that makes it true). | **Be Able to Do**   * solve one-step equations given in concrete or pictorial form. * represent a problem in context concretely or pictorially and solve the problem. * solve one-step equations involving symbols for the unknown. * write and solve one-step equations for problems in context. |

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|  | * in order to preserve the equality or balance in an equation, one side of the equation cannot be changed without performing the same change to the other side of the equation. * mathematical symbols can be used to represent problems as equations. * guess and test is a valid method of solving equations, although it can be time consuming. * the sides of an equation can be switched and the equation remains the same (a = b so b = a). * the symbol for the unknown can be written on either side of an equation. * there are different ways to write the same equation. | * write a problem in context that a given equation represents. * verify a solution to an equation. * explain what an equation tells them. |
| **Key Questions:** [**Top**](#top) | | |
| * Why would a symbol be used for an unknown quantity? * Why must both sides of an equation be changed in the same way? * Why would equations be written? * What strategies do you use when solving an equation? * What does the solution of an equation represent? * How do you know if a solution to an equation is correct? | | |

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| **Suggestions for assessment:** [**Top**](#top) |
| **Big Idea:**  Representing equations.  **Suggestions for assessment tasks:**   1. Have the students write an equation for a problem and explain the meaning of the symbol used. 2. Give the students an equation and ask them to give a context that the equation could be used to describe.   **What to look for:**   * See [*Representing Equations Rubric*](file:///C:\Users\ru593\AppData\Local\Temp\Representing%20Equations%20Rubric.doc)*.*   **Big Idea:**  Preservation of equality.  **Suggestions for assessment tasks:**   1. Have the students explain the meaning of the solution to an equation. 2. Have the students explain the strategies and reasoning used in solving an equation. 3. Provide the students with a solution to the equation and ask them to determine if the solution is correct, and if it isn’t to identify the error that was made.   **What to look for:**   * See [*Preservation of Equality Rubric.*](file:///C:\Users\ru593\AppData\Local\Temp\Preservation%20of%20Equality%20Rubric.doc) |

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| **Suggestions for instruction:** [**Top**](#top) |
| **Big Idea:**  Representing equations.  **Suggestions for instructional activities:**   1. Do a concept attainment activity with equations involving an unknown value. The examples and counter examples should highlight the difference between equations and expressions, and equations and numerical statements within an equation form (e.g., 3 x 8 = 24). Also be sure to include equations and expressions that involve all four of the operations the students know (addition, subtraction, multiplication and division). Begin by giving the students examples and non-examples, asking them to think about the characteristics that they believe are determining whether something given will be a yes or no. After a few examples and non-examples, show the students a new item and ask them where they think you should place it (as a Yes or No). Have the students respond by giving thumbs up for yes, thumbs down for no, and thumbs sideways for not sure. If there is disagreement, or many students who are not sure, put those items into a Maybe pile for future reflection. If the students’ decision is wrong, still put it in the column they have chosen. Gradually, start asking the students to share their reasoning and the characteristics that they believe to be defining the yes column. Allow the students to discuss each other’s ideas, and if you notice that they have a characteristic wrong, provide an example or non-example (that you decide on its placement) that will contradict their thinking. Allow the students to change items around and put the Maybes into Yes or No as they become more confident of how the Yes category is defined. Make a summary list of the characteristics that they are giving, and then tell them that what they have defined is called an equation with one unknown. Ask the students to write their own definition, including examples and non-examples, in their journal. 2. Ask the students what they think the unknown stands for. Provide them with a series of very simple equations, involving different operations. Make sure that the unknown occurs on different sides of the equation, as a single term and as either the first or second term. At first, allow them to say that it is 10 or 5 – the actual solution to the equation. Then, have the students notice that the unknown seems to have different values and ask them why that is. Gradually get into the discussion about the unknown representing a value that isn’t known, but can be determined. 3. Provide the students with problems and contexts and ask them to write equations that represent the problems or context. Have the students explain the strategies used. Also, have the students share their equations, highlighting any that are different (but actually represent the same problem (adding six to one side of the equation, or subtracting six to the other side). Keep these equations on hand for when the students start to solve the equations so that they can determine if they are equivalent equations.   **Big Idea:**  Preservation of equality.  **Suggestions for instructional activities:**  *Because of the obvious nature of answers to one-step equations, students will usually just “get” the answer. It is important that they explain how they “get” it, or if they are using a guess and test strategy (which is very good) they need to be able to explain which values they guess are correct and which are not.*   1. Have the students discuss what the equal sign means: that there is a balance in quantity between the two sides of the equation. Because the students have not yet learned about integers, it is very important to not give them any questions that would end up with a solution less than zero. As well, as they begin solving the equations, be careful to not use terminology such as adding a negative value. For grade four, it is important to refer to this as subtraction. If students describe it in the other terms naturally, this is okay, but be sure to reiterate their comments in terms of subtraction so that the students aren’t expected to understand negative numbers. 2. Have the students model what they believe it means for the two sides of the equation to be balanced. Start by using equations that involve numbers only – such as 45 + 9 = 54, and have the students confirm that the two sides are equal. Ideally, a commercial balance set is best in enforcing the idea of balance. Next, have the students start doing different things to one side of the equation, (adding, subtracting, multiplying or dividing by a value) and have them compare if the two sides are not equal. Once they realize that they aren’t, have the students discuss why they think they are not equal, and what they would need to do to have the equality be maintained. Have the students verify their ideas by representing the actions they have decided upon and checking that the two sides are still balanced. Have the students write about maintaining balance in an equation in their journals. 3. Have the students work with concrete and pictorial representations of equations representing contexts or problems, focusing on maintaining the balance between the two sides of the equations. As the students develop an understanding of maintaining of balance and are confident in strategies they can apply, gradually work on having the students represent their work abstractly by recording symbolically each part of their concrete or pictorial representation. Encourage a lot of discussion and sharing of ideas and understandings. Often, students will select the inverse operation in the process of solving an equation. Allow this to occur, and encourage the students to reflect on the impact of their choice so that they develop an understanding of how to make choices in solving equations. The purely symbolic representation of their solution strategies is the least important aspect of this outcome. 4. Provide the students with a problem and a series of equations. Ask the students to identify which equations represent the problem and how they have come to their decision. Be sure to include multiple correct possible equations because some students will represent the same problem using different equations (although all equivalent). For example, if a problem says that one person is 14 years older than the other, some students will see that as a difference of the two ages, some will see it as the older age minus 14 being the younger age, and some will see it as the younger age plus 14 being the older age. Also be sure to include incorrect representations, such as the younger age minus the older age equalling 14. It is very important that the students discuss their choices and provide their reasoning so   that alternate interpretations and errors in reasoning can be uncovered and understood rather than just provided or corrected.   1. Provide the students with an equation and ask the students to rewrite the equation in different but equivalent forms. Have the students discuss their reasoning. This should not be about solving the equation, but rather about understanding what the equation represents and how else that can be represented. 2. Provide the students with an equation and series of problems that involve the same number values. Ask the students to identify those problems that the equation represents. Have the students discuss the strategies that they used. Some students may do this completely abstractly, while others may use concrete or pictorial representations to understand the equation and/or the problems. Both strategies are valid and the students should all be exposed to the alternate strategies that emerge in the classroom. 3. In pairs, have the students select a context or topic of interest to them. Next, provide the students with an equation and ask them to come up with a problem based on a context the equation represents. If any of the pairs are finding their context difficult to work with, allow them to change contexts. Have the students share their problems and discuss the strategies they used along with any difficulties they encountered. 4. Have the students continue to work on solving equations, solving problems, and writing problems for given equations to internalize their developing understandings of equality and equations. 5. Throughout the learning process, have the students constantly reflect on the meaning of the solutions to the equations they are working with, what the equal sign tells them, and what the unknown symbol represents. Encourage the students to verify their solutions in order to develop their understanding of the solution (rather than to just “check” if they got the right answer). Monitor the students understanding of verification of solutions – they should not be putting a lone equal sign in the equation when verifying. Either each side of the equation should be simplified separately and then a note made as to whether the two sides are equal, or if the equal sign is included a question mark could be placed over it to indicate that the equality statement is not yet verified. Watch that the students do not move terms from one side of the equation to the other while verifying. Have the students discuss why this is not a correct strategy (because you must assume equality to do so). |