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Daryl Bangsund
Good Spirit School Division
LEADS

Dr. Murray Bremner
Department of Mathematics and Statistics
University of Saskatchewan

Linda Goulet
Associate Professor
First Nations University of Canada

Angie Harding
Regina Roman Catholic Separate School Division
Saskatchewan Teachers' Federation

Susan Jeske
Prairie Spirit School Division
Saskatchewan Teachers' Federation

Wendy Lang
St. Paul's Roman Catholic Separate School Division
Saskatchewan Teachers' Federation

George McHenry
Division Board Trustee
Saskatchewan School Boards Association

Dr. Shaun Murphy
College of Education
University of Saskatchewan

Dr. Kathy Nolan
Faculty of Education
University of Regina

Kathi Sandbeck
Sun West School Division
Saskatchewan Teachers' Federation

Doug Sthamann
Regina Public School Division
Saskatchewan Teachers' Federation

Rodney White
North East School Division
Saskatchewan Teachers' Federation

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Introduction

As a required area of study, Mathematics is to be allocated 200 minutes per week for the entire school year at Grade 8. It is important that students receive the full amount of time allocated to their mathematical learning and that the learning be focused upon students attaining the understanding and skills as defined by the outcomes and indicators stated in this curriculum.

Outcomes describe the knowledge, skills, and understandings that students' are expected to attain by the end of a particular grade level.

The outcomes in Grade 8 Mathematics build upon students' prior learnings and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These continuing learnings prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts.

Indicators are a representative list of the types of things a student should know or be able to do if they have attained the outcome.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. These indicators are a representative list of the kinds of things a student needs to know and/or be able to do in order to achieve the learnings intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can and should be created by teachers to meet the needs and circumstances of their students and communities.

This curriculum's outcomes and indicators have been designed to address current research in mathematics education as well as the needs of Saskatchewan students. The Grade 8 Mathematics outcomes have been influenced by the renewal of the Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K-9 Mathematics* outcomes (2006). Changes throughout all of the grades have been made for a number of reasons including:

- decreasing content in each grade to allow for more depth of understanding
- rearranging concepts to allow for greater depth of learning in one year and to align related mathematical concepts
- increasing the focus on numeracy (i.e., understanding numbers and their relationship to each other) beginning in Kindergarten
- introducing algebraic thinking earlier.

Also included in this curriculum is information regarding how Grade 8 Mathematics connects to the K-12 goals for mathematics. These goals define the purpose of mathematics education for Saskatchewan students.

In addition, teachers will find discussions of the critical characteristics of mathematics education, assessment and evaluation of student learning in mathematics, inquiry in mathematics, questioning in mathematics, and connections between Grade 8 Mathematics and other Grade 8 areas of study within this curriculum.

Finally, the Glossary provides explanations of some of the mathematical terminology you will find in this curriculum.

Core Curriculum

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its various components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* (August 2007) on the Ministry of Education website.

Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. K-12 Mathematics contributes to the Goals of Education through helping students achieve knowledge, skills, and attitudes related to these Broad Areas of Learning.

Building Lifelong Learners

Related to the following Goals of Education:

- *Basic Skills*
- *Lifelong Learning*
- *Self Concept Development*
- *Positive Lifestyle*

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that will support their learning of new mathematical concepts and applications that may be encountered within both career and personal interest choices. Students who successfully complete their study of K-12 Mathematics should feel confident about their mathematical abilities and have developed the knowledge, understandings, and abilities necessary to make future use and/or studies of mathematics meaningful and attainable.

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

(NCTM, 2000, p. 20)

In order for mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 Mathematics). It is crucial that the students discover the mathematics outlined in the curriculum rather than the teacher covering it.

Building a Sense of Self and Community

Related to the following Goals of Education:
- *Understanding & Relating to Others*
- *Self Concept Development*
- *Positive Lifestyle*
- *Spiritual Development*

To learn mathematics with deep understanding, students not only need to interact with the mathematical content, but with each other as well. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue are exposed to a wide variety of perspectives and strategies from which to construct a sense of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of the mathematics. At the same time, students also learn to respect and value the contributions of others.

Many of the topics and problems in a mathematics classroom can be initiated by the children themselves. In a classroom focused on working mathematically, teachers and children work together as a community of learners; they explore ideas together and share what they find. It is very different to the traditional method of mathematics teaching, which begins with a demonstration by a teacher and continues with children practicing what has been demonstrated.
(Skinner, 1999, p. 7)

Mathematics also provides many opportunities for students to enter into communities beyond the classroom by engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students will develop their personal and social identity, and learn healthy and positive ways of interacting and working together with others.

Building Engaged Citizens

Related to the following Goals of Education:
- *Understanding & Relating to Others*
- *Positive Lifestyle*
- *Career and Consumer Decisions*
- *Membership in Society*
- *Growing with Change*

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students such as trends in global warming, homelessness, technological health issues (oil spills, hearing loss, carpal tunnel syndrome, diabetes), and discrimination can be used to engage the students in interacting and contributing positively to their classroom, school, community, and world. With the understandings that students can derive through mathematical analysis, they become better informed and have a greater respect for, and understanding of, differing opinions and possible options. With these understandings, students can make better informed and more personalized decisions regarding roles within, and contributions to, the various communities in which students are members.

Cross-curricular Competencies

The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater.
(NCTM, 2000, p. 4)

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes which are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

Developing Thinking

K-12 Goals
- *thinking and learning contextually*
- *thinking and learning creatively*
- *thinking and learning critically.*

It is important that, within their study of mathematics, students are engaged in personal construction and understanding of mathematical knowledge. This most effectively occurs through student engagement in inquiry and problem solving when students are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which students are asked to consider questions such as "what would happen if …", "could we find …", and "what does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool which can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution..

Developing Identity and Interdependence

K-12 Goals
- *understanding, valuing, and caring for oneself*
- *understanding, valuing, and respecting human diversity and human rights and responsibilities*
- *understanding and valuing social and environmental interdependence and sustainability.*

Given an appropriate learning environment in mathematics, students can develop both their self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It can also help students take an active role in defining and maintaining the classroom environment and accept responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning serves to support students in behaving respectfully towards themselves and others.

Developing Literacies

Through their mathematics learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be regularly engaged in exploring a variety of representations for mathematical concepts and should be expected to communicate in a variety of ways about the mathematics being learned. An important part of learning mathematical language is to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. The study of mathematics should encourage the appropriate use of technology. Moreover, students should be aware of and able to make the appropriate use of technology in mathematics and mathematics learning. It is important to encourage students to use a variety of forms of representation (concrete manipulatives, physical movement, oral, written, visual, and symbolic) when exploring mathematical ideas, solving problems, and communicating understandings. All too often, it is assumed that symbolic representation is the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper students' understanding becomes.

Students gain insights into their thinking when they present their methods for solving problems, when they justify their reasoning to a classmate or teacher, or when they formulate a question about something that is puzzling to them. Communication can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols. Misconceptions can be identified and addressed. A side benefit is that it reminds students that they share responsibility with the teacher for the learning that occurs in the lesson.

(NCTM, 2000, pp. 60 – 61)

K-12 Goals
- *constructing knowledge related to various literacies*
- *exploring and interpreting the world through various literacies*
- *expressing understanding and communicating meaning using various literacies.*

Ideas are the currency of the classroom. Ideas, expressed by any participant, warrant respect and response. Ideas deserve to be appreciated and examined. Examining an idea thoughtfully is the surest sign of respect, both for the idea and its author.

(Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, Human, 1997, p. 9)

Developing Social Responsibility

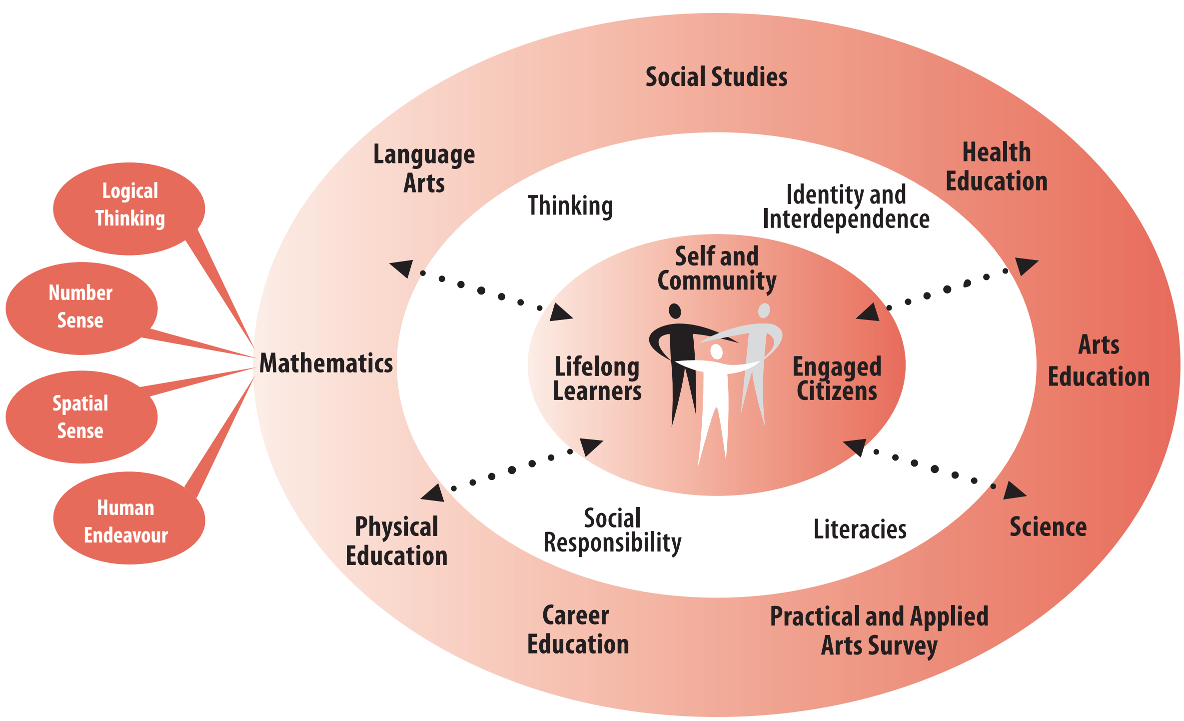
As students progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment be co-constructed by the teacher and students to support respectful, independent,

K-12 Goals
- *using moral reasoning*
- *engaging in communitarian thinking and dialogue*
- *contributing to the well-being of self, others, and the natural world.*

and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for situational validity, and strengthened.

Aim and Goals of K-12 Mathematics

The aim of the K-12 mathematics program is to prepare individuals who value mathematics and appreciate its role in society. The K-12 Mathematics curricula are designed to prepare students to cope confidently and competently with everyday situations that demand the use of mathematical concepts including interpreting quantitative information, estimating, performing calculations mentally, measuring, understanding spatial relationships, and problem solving. The Mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.



Defined below are four goals for K-12 Mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes must, therefore, also promote student achievement with respect to the goals.

Logical Thinking

Through their learning of K-12 Mathematics, students should **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- inductive and deductive thinking
- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships
- modeling and representing (including concrete, oral, physical, pictorial, and symbolical representations)
- conjecturing and asking "what if" (mathematical play).

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of and connections between multiple representations.

A . . . feature of the social culture of [mathematics] classrooms is the recognition that the authority of reasonability and correctness lies in the logic and structure of the subject, rather than in the social status of the participants. The persuasiveness of an explanation, or the correctness of a solution depends on the mathematical sense it makes, not on the popularity of the presenter.

(Hiebert et al., 1997, p. 10)

Number Sense

Through their learning of K-12 Mathematics, students should **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations

In Grade 8, students are beginning to study square roots. What other irrational numbers have students encountered in their study of mathematics and how do students understand these numbers within the context of other numbers (whole numbers, fractions and decimals) studied previously?

(including concrete, oral, physical, pictorial, and symbolical representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 Mathematics, students should **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

As students sort, build, draw, model, trace, measure, and construct, their capacity to visualize geometric relationships will develop.
(NCTM, 2000, p. 165)

Development of a strong spatial sense requires students to have ongoing experiences with:
- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

Many Grade 8 outcomes develop students' spatial sense in terms of 3-D objects, 2-D shapes, Pythagorean Theorem, tessellations, and data displays. How can you assess students spatial understanding of these concepts?

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation of conjectures based upon patterns that are discovered and tested should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

Mathematics as a Human Endeavour

Through their learning of K-12 Mathematics, students should **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematically and personally)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics

What types of instructional strategies support student attainment of the K-12 Mathematics goals?

How can student attainment of these goals be assessed and the results be reported?

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet their particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

Meaning does not reside in tools; it is constructed by students as they use tools.
(Hiebert et al., 1997, p. 10)

The content found within the grade level outcomes for the K-12 Mathematics programs, and its applications, is first and foremost the vehicle through which students can achieve the four goals of K-12 Mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Teaching Mathematics

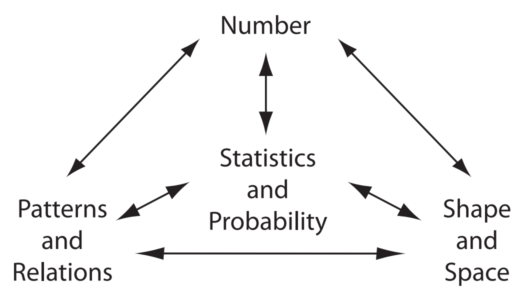
At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula there is very little to cover, but an awful lot to uncover [discover]." This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students this is a marked change from the way mathematics was taught to them. Research and experience indicate there is a complex, interrelated set of characteristics that teachers need to be aware of in order to provide an effective mathematics program.

Critical Characteristics of Mathematics Education

The following sections in this curriculum highlight some of the different facets for teachers to consider in the process of changing from covering to supporting students in discovering mathematical concepts. These facets include the organization of the outcomes into strands, seven mathematical processes, the difference between covering and discovering mathematics, the development of mathematical terminology, the continuum of understanding from the concrete to the abstract, modelling and making connections, the role of homework in mathematics, and the importance of ongoing feedback and reflection.

Strands

The content of K-12 Mathematics can be organized in a variety of ways. In this curriculum, the outcomes and indicators are grouped according to four strands: **Number, Patterns and Relations, Shape and Space, and Statistics and Probability.**



Although this organization implies a relatedness among the outcomes identified in each of the strands, it should be noted the mathematical concepts are interrelated across the strands as well as within strands. Teachers are encouraged to design learning activities that integrate outcomes both within a strand and across the strands so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate strands.

Mathematical Processes

This Grade 8 Mathematics curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in K-12 Mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. Teachers should consider carefully in their planning those processes indicated as being important to supporting student achievement of the various outcomes.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links between their own language, ideas, and prior knowledge, the formal language and symbols of mathematics, and new learnings.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding for that terminology.

Concrete, pictorial, symbolic, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Communication works together with reflection to produce new relationships and connections. Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics.
(Hiebert et al., 1997, p. 6)

> *Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding …. Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching.*
> *(Caine & Caine, 1991, p.5)*

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase student willingness to participate and be actively engaged.

> *By Grade 8, students should have many different computational and estimation strategies within their mathematical repertoire. It is important to have students share and explain these strategies, and to encourage the students to see the relationships between these strategies and possible strategies for new computations being learned in Grade 8.*

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

> *Mathematical problem-solving often involves moving backwards and forwards between numerical/algebraic representations and pictorial representations of the problem.*
> *(Haylock & Cockburn, 2003, p. 203)*

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you …?", "Can you …?", or "What if …?", the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for, and engage in finding, a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. High-order inquiry challenges students to think and develop a sense of wonder about mathematics.

By Grade 8, students' mathematical reasoning can be quite sophisticated. How can you assess the students' use of mathematical reasoning?

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Posing conjectures and trying to justify them is an expected part of students' mathematical activity.
(NCTM, 2000, p. 191)

Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

[Visualization] involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.
(Armstrong, 1993, p.10)

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes including aspects such as dimensions and measurements.

How will you know if, and how, your Grade 8 students are visualizing a mathematical concept?

Visualization is also important in the students' development of abstraction and abstract thinking and reasoning. Visualization provides a connection between the concrete, physical, and pictorial to the abstract symbolic. Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations as well as the use of communication to develop connections between different contexts, content, and representations.

Technology [T]

Technology tools contribute to student achievement of a wide range of mathematical outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology should not be used as a replacement for basic understandings and intuition.
(NCTM, 2000, p. 25)

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what needs to be covered and what can be discovered is crucial in planning for mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols.

What mathematical content in Grade 8 can students uncover (through the careful planning of a teacher) and what does a teacher need to tell the students?

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as procedures and strategies, rules, and problem solving. Any learning in mathematics that is a result of the logical structure of mathematics can and should be constructed by students.

For example, in Grade 8, the students encounter linear relations outcome P8.1:

> Demonstrate an understanding of linear relations concretely, pictorially (including graphs), physically, and symbolically. [CN, ME, PS, R, T, V]

In this outcome, the term "linear relation" is a social convention of the mathematics the students are learning and, as such, it is something that the teacher must tell the student. What the students learn about linear relations, such as recognizing linear relations in different forms of representations and creating different representations for the same linear relation, should emerge from the students' analysis of patterns through exploring, conjecturing, verifying, and generalizing to more abstract forms. This type of learning requires students to work concretely, physically, orally, pictorially, in writing, and symbolically. It also requires that students share their ideas with their classmates and reflect upon how the ideas and understandings of others relate to, inform, and clarify what students individually understand. In this type of learning, the teacher does not tell the students how to do the mathematics but, rather, invites the students to explore and develop an understanding of the logical structures inherent in the mathematics in increasing patterns. Thus, the teacher's role is to create inviting and rich inquiring tasks and to use questioning to effectively probe and further students' learning.

> *"The plus sign, for example, and the symbols for subtraction, multiplication, and division are all arbitrary convention. … Learning most of mathematics, however, relies on understanding its logical structures. The source of logical understanding is internal, requiring a child to process information, make sense of it, and figure out how to apply it."*
> *(Burns and Sibley, 2000, p. 19)*

Development of Mathematical Terminology

Part of learning mathematics is learning how to speak mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that students already know or that make sense to them.

> *Teachers should model appropriate conventional vocabulary.*
> *(NCTM, 2000, p. 131)*

For example, in outcome SS8.1:

> Demonstrate understanding of the Pythagorean Theorem concretely or pictorially, and symbolically, and by solving problems. [CN, PS, R, T, V]]

the term “Pythagorean Theorem” should not be introduced and defined for the student before the student has had an opportunity to explore the relationship between the area of squares with side lengths equal to that of the lengths of the legs of triangles. Through these explorations, students would notice that, when the triangle is a right triangle, there is a specific relationship between the areas. When the students are able to describe their understandings of this relationship, then the actual mathematical terminology (Pythagorean Theorem) is best introduced because the students have conceptual knowledge with which to connect the new words.

Upon introducing the new terms, the teacher should also be checking if the students have other connections to the new words in non-mathematical settings. For example, in Grade 8 Mathematics, students learn about the constructing of nets for 3-D objects. Many students will have other contexts for understanding the term "nets". It is important for the students to discuss and compare the contexts for the term and its meaning.

The Concrete to Abstract Continuum

It is important that, in learning mathematics, students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a concrete starting point. Therefore, the teacher must be prepared to engage students at different points along the continuum.

It is important for students to use representations that are meaningful to them.
(NCTM, 2000, p. 140)

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. For example, when considering a problem about the total number of pencils, some students might find it more concrete to use pictures of pencils as a means of representing the situation. Other students might find coins more concrete because they directly associate money with the purchasing or having of a pencil.

As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following problem involving ratios:

> To make a particular shade of green, Darius determined that he needed to use two parts of blue paint to three parts of yellow paint. If Darius wants to make 1L of green paint, how many mL of blue and yellow paint will he need to mix?

Depending upon how the problem is expected to be solved (or if there is any specific expectation), this problem can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives, pictures, role play), or both.

Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and symbolic). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another (how could you write what you've done here using mathematical symbols?) or by having students compare their representations with others around the class.

A major responsibility of teachers is to create a learning environment in which students' use of multiple representations is encouraged.
(NCTM, 2000, pp. 139)

In making these connections, students should also be asked to reflect upon the mathematical ideas and concepts that students already know are being used in their new models (e.g., I know that addition means to put things together into a group, so I'm going to move the two sets of blocks together to determine the sum).

Making connections also involves looking for patterns. For example, in outcome SS8.3:

> Demonstrate understanding of volume limited to right prisms and cylinders (concretely, pictorially, or symbolically) by:
> - relating area to volume
> - generalizing strategies and formulae
> - analyzing the effect of orientation
> - solving problems.
>
> [CN, PS, R, V]

students can explore how volume is related to area, resulting in the students making connections between the area of the base of a right prism or cylinder and its volume. When students determine these relationships and connections for themselves, the formulae for the volumes of right prisms and cylinders become logical and obvious to the students.

Characteristics of Good Math Homework

- *It is accessible to children at many levels.*
- *It is interesting both to children and to any adults who may be helping.*
- *It is designed to provoke deep thinking.*
- *It is able to use concepts and mechanics as means to an end rather than as ends in themselves.*
- *It has problem solving, communication, number sense, and data collection at its core.*
- *It can be recorded in many ways.*
- *It is open to a variety of ways of thinking about the problem although there may be one right answer.*
- *It touches upon multiple strands of mathematics, not just number.*
- *It is part of a variety of approaches to and types of math homework offered to children throughout the year.*

(Raphel, 2000, p. 75)

Role of Homework

The role of homework in teaching for deep understanding is very important and also quite different from homework that is traditionally given to students. Students should be given unique problems and tasks that help students to consolidate new learnings with prior knowledge, explore possible solutions, and apply learnings to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of the drill will vary among different learners. In addition, when used as homework, drill and practice frequently serves to cause frustration, misconceptions, and boredom to arise in students.

As an example of the type or style of homework that can be used to help students develop deep understanding of Grade 8 Mathematics, consider outcome SS8.4:

Demonstrate an understanding of tessesllation by:
- explaining the properties of shapes that make tessellating possible
- creating tessellations
- identify tessellations in the environment.

[C, CN, R, V]

When students realize that if a 2-D shape has an internal angle that is a factor of 360° it will tessellate, a homework task to explore what pairs of shapes will tessellate can be given. This task could be differentiated for different students, ranging from a specific set of tessellating shapes, to any shapes the student might want to consider, to a set of shapes that includes ones that cannot tessellate on their own. The results of the students' explorations can then be shared and discussed in class, leading the students to a deeper understanding of how tessellations can occur and how to select shapes so that a tessellation is possible.

Ongoing Feedback and Reflection

Feedback can take many different forms. Instead of saying, "This is what you did wrong," or "This is what you need to do," we can ask questions: "What do you think you need to do? What other strategy choices could you make? Have you thought about…?"

(Stiff, 2001, p. 70)

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information in the teacher's planning for further and future learnings.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals.

Not all feedback has to come from outside – it can come from within. When adults assume that they must be the ones who tell students whether their work is good enough, they leave them handicapped, not only in testing situations (such as standardized tests) in which they must perform without guidance, but in life itself.
(NCTM, 2000, p. 72)

Teaching for Deep Understanding

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. As an example, the square root sign ($\sqrt{a}$) is something which the teacher must introduce and ensure that students know. It is the symbol used to represent the process of finding the quantity (or the quantity itself) that when squared is a. The meaning, purpose, and computation of square roots, however; are learnings that should be discovered through the students' investigation of patterns, relationships, abstractions, and generalizations. It is important for teachers to reflect upon outcomes to identify what students need to know, understand, and be able to do. Opportunities must be provided for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

It is important that a mathematics learning environment include effective interplay of:

- reflection
- exploration of patterns and relationships
- sharing of ideas and problems
- consideration of different perspectives
- decision making
- generalizing
- verifying and proving
- modeling and representing.

A simple model for talking about understanding is that to understand something is to connect it with previous learning or other experiences… A mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences, and pictures.
(Haylock & Cockburn, 2003, p. 18)

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

Inquiry

Inquiry is a philosophical stance rather than a set of strategies, activities, or a particular teaching method. As such, inquiry promotes intentional and thoughtful learning for teachers and children.
(Mills & Donnelly, 2001, p. xviii)

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

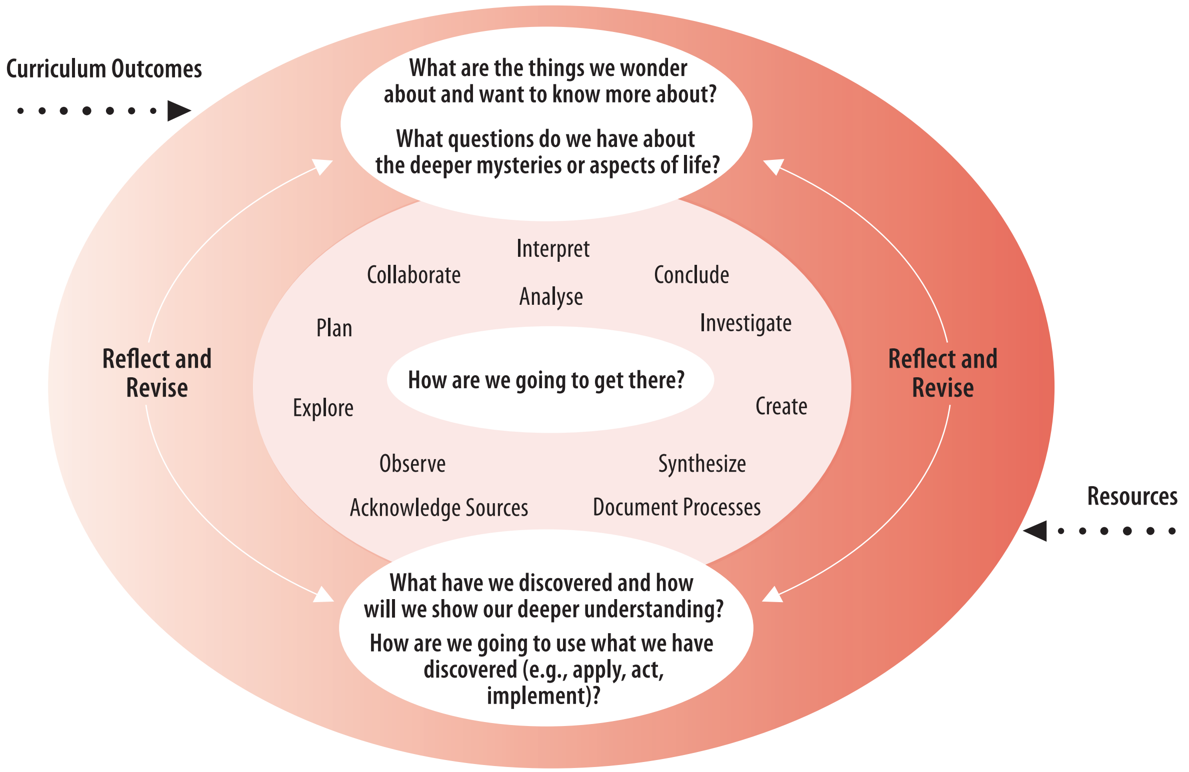
- construct deep knowledge and deep understanding rather than passively receiving it
- are directly involved and engaged in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understandings
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning and mastery of curriculum content and skills.

(Adapted from Kuhlthau & Todd, 2008, p. 1)

What topics might Grade 8 students be curious about, and how can these topics be connected to the students' learning of mathematics?

Inquiry learning is not a step-by-step process, but rather a cyclical process, with various phases of the process being revisited and rethought as a result of students' discoveries, insights, and co-construction of new knowledge. The following graphic shows various phases of this cyclical inquiry process.

Constructing Understanding Through Inquiry



Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers will move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, conclude, document, reflect on learning, and develop new questions for further inquiry.

In Mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students too must understand this difference.

Something is only a problem if you don't know how to get from where you are to where you want to be. Be sure that Grade 8 students are solving such problems.

Questions may be one of the most powerful technologies invented by humans. Even though they require no batteries and need not be plugged into the wall, they are tools which help us make up our minds, solve problems, and make decisions. – Jamie McKenzie (Schuster & Canavan Anderson, 2005, p. 1)

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more curriculum entry points; however, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry and give students direction for discovering deep understandings about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions:

- *cause genuine and relevant inquiry into the important ideas and core content.*
- *provide for thoughtful, lively discussion, sustained inquiry, and new understanding as well as more questions.*
- *require students to consider alternatives, weigh evidence, support their ideas, and justify their answers.*
- *stimulate vital, ongoing rethinking of key ideas, assumptions, and prior lessons.*
- *spark meaningful connections with prior learning and personal experiences.*
- *naturally recur, creating opportunities for transfer to other situations and subjects.*

(Wiggins & McTighe, 2005, p. 110)

Effective questions in Mathematics are the key to initiating and guiding students' investigations and critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "When would you want to add two numbers less than 100?"
- "How do you know you have an answer?"
- "Will this work with every number? Every similar situation?"
- "How does your representation compare to that of your partner?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning and should be an integral part of planning in mathematics. Questioning should also be used to encourage students to reflect on the inquiry process and the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

- *help students make sense of the mathematics.*
- *are open-ended, whether in answer or approach. There may be multiple answers or multiple approaches.*
- *empower students to unravel their misconceptions.*
- *not only require the application of facts and procedures but encourage students to make connections and generalizations.*
- *are accessible to all students in their language and offer an entry point for all students.*
- *lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.*

(Schuster & Canavan Anderson, 2005, p. 3)

As teachers of mathematics, we want our students not only to understand what they think but also to be able to articulate how they arrived at those understandings.
(Schuster & Canavan Anderson, 2005, p. 1)

Reflection and Documentation of Inquiry

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics may take the form of reflective journals, notes, drafts, models, and works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understandings.

It is important that students are required and allowed to engage in the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen student understanding.

When we ask good questions in math class, we invite our students to think, to understand, and to share a mathematical journey with their classmates and teachers alike. Students are no longer passive receivers of information when asked questions that challenge their understandings and convictions about mathematics. They become active and engaged in the construction of their own mathematical understanding and knowledge.
(Schuster & Canavan Anderson, 2005, p. 1)

Outcomes and Indicators

Number Strand	
Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour	
Outcomes (What students are expected to know and be able to do.)	**Indicators** (Students who have achieved this outcome should be able to:)
***N8.1** Demonstrate understanding of the square and principle square root of whole numbers concretely or pictorially and symbolically. [CN, ME, R, T, V]*	a. Recognize, show, and explain the relationship between whole numbers and their factors using concrete or pictorial representations (e.g., using a set number of tiles, create rectangular regions and record the dimensions of those regions, and describe how those dimensions relate to the factors of the number). b. Infer and verify relationships between the factors of a perfect square and the principle square root of a perfect square. c. Determine if specific numbers are perfect squares through the use of different types of representations and reasoning, and explain the reasoning. d. Describe and apply the relationship between the principle square roots of numbers and benchmarks using a number line. e. Explain why the square root of a number shown on a calculator may be an approximation. f. Apply estimation strategies to determine approximate values for principle square roots. g. Determine the value or an approximate value of a principle square root with or without the use of technology. h. Identify a number with a principle square root between two given numbers and explain the reasoning. i. Share the story, in writing, orally, drama, dance, art, music, or other media, of the role and significance of square roots in a personally selected historical or modern application situation (e.g., Archimedes and the square root of 3, Pythagoras and the existence of square roots, role of square roots in Pythagoras' theorem, use of square roots in determining dimensions of a square region from the area, use of square roots to determine measurements in First Nations beading patterns, use of square roots to determine dimensions of nets).

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
N8.2 Expand and demonstrate understanding of percents greater than or equal to 0% (including fractional and decimal percents) concretely, pictorially, and symbolically. [CN, PS, R, V]	a. Recognize, represent, and explain situations, including for self, family, and communities, in which percents greater than 100 or fractional percents are meaningful (e.g., the percent profit made on the sale of fish). b. Represent a fractional percent and/or a percent greater than 100 using grid paper. c. Describe relationships between different types of representation (concrete, pictorial, and symbolic in percent, fractional, and decimal forms) for the same percent (e.g., how do 345 coloured grid squares relate to 345%, or why is 345% the same as 3.45). d. Record the percent, fraction, and decimal forms of a quantity shown by a representation on grid paper. e. Apply understanding of percents to solve problems, including situations involving combined percents or percents of percents (e.g., PST + GST, or 10% discount on a purchase already discounted 30%) and explain the reasoning. f. Explain, using concrete, pictorial, or symbolic representations, why the order of consecutive percents does not impact the final value (e.g., a decrease of 15% followed by an increase of 5% results in the same quantity as an increase of 5% followed by a decrease of 15%). g. Demonstrate, using concrete, pictorial, or symbolic representations, that two consecutive percents applied to a specific situation cannot be added or subtracted to give an overall percent change (e.g., a population increase of 10% followed by a population increase of 15% is not a 25% increase, a decrease of 10% followed by an increase of 10% will result in an overall change). h. Analyze choices and make decisions based upon percents and personal or community concerns and issues (e.g., deciding whether or not to have surgery if given a 75% chance of survival, deciding how much to buy if you can save 25% when two items are purchased, deciding whether or not to hunt for deer when a known percent of deer have chronic wasting disease, deciding about whether or not to use condoms knowing that they are 95% effective as birth control, making decisions about diet knowing that a high percentage of Aboriginal peoples have or will get diabetes).

Outcomes	Indicators
N8.2 (continued)	i. Explain the role and significance of percents in different situations (e.g., polls during elections, medical reports, percent down on purchases). j. Pose and solve problems involving percents stated as a percent, fraction, or decimal quantity.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
N8.3 Demonstrate understanding of rates, ratios, and proportional reasoning concretely, pictorially, and symbolically. ***[C, CN, PS, R, V]***	Note: It is important that ratios and rates be studied within the context of measurement and as types of patterns and relations. a. Identify and explain ratios and rates in familiar situations (e.g., cost per music download, traditional mixtures for bleaching, time for a hand-sized piece of fungus to burn, mixing of colours, number of boys to girls at a school dance, rates of traveling such as car, skidoo, motor boat or canoe, fishing nets and expected catches, or number of animals hunted and number of people to feed). b. Identify situations (such as providing for the family or community through hunting) in which a given quantity of $\frac{a}{b}$ represents a: • fraction • rate • quotient • percent • probability • ratio. c. Demonstrate (orally, through arts, concretely, pictorially, symbolically, and/or physically) the difference between ratios and rates.

Outcomes	Indicators
N8.3 (continued)	d. Verify or contradict proposed relationships between the different roles for quantities that can be expressed in the form $\frac{a}{b}$. For example: • a rate cannot be represented by a percent because a rate compares two different types of measurements while a percent compares two measurements of the same type • probabilities cannot be used to represent ratios because probabilities describe a part to whole relationship but ratios describe a part to part relationship • a fraction is not a ratio because a fraction represents part to whole • a ratio cannot be written as a fraction, unless the quantity of the whole is first determined (e.g., 2 parts white and 5 parts red paint is $\frac{2}{7}$ white) • a ratio cannot be written as percent unless the quantity of the whole is first determined (e.g., a ratio of 4 parts blue and 6 parts red paint can be described as having 40% blue). e. Write the symbolic form (e.g., 3:5 or 3 to 5 as a ratio, $3/min or $3 per one minute as a rate) for a concrete, physical, or pictorial representation of a ratio or rate. f. Explain how to recognize whether a comparison requires the use of proportional reasoning (ratios or rates) or subtraction. g. Create and solve problems involving rates, ratios, and/or probabilities.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
N8.4** **Demonstrate understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. [C, CN, ME, PS]	Note: It is intended that positive fractions be interpreted to include both whole numbers and mixed numbers as well as common fractions a. Identify and describe situations relevant to self, family, or community in which multiplication and division of fractions are involved. b. Model the multiplication of two positive fractions and record the process symbolically. c. Compare the multiplication of positive fractions to the multiplication of whole numbers, decimals, and integers. d. Generalize and apply strategies for determining estimates of products of positive fractions

Outcomes

N8.4 (continued)

Indicators

e. Generalize and apply strategies for multiplying positive fractions.

f. Critique the statement "Multiplication always results in a larger quantity" and reword the statement to capture the points of correction or clarification raised (e.g., $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ which is smaller than $\frac{1}{2}$).

g. Explain, using concrete or pictorial models as well as symbolic reasoning, how the distributive property can be used to multiply mixed numbers. For example, $2\frac{1}{2} \times 3\frac{1}{4} = \left(2 + \frac{1}{2}\right) \times \left(3 + \frac{1}{4}\right) = (2 \times 3) + \left(2 \times \frac{1}{4}\right) + \left(\frac{1}{2} \times 3\right) + \left(\frac{1}{2} \times \frac{1}{4}\right)$.

h. Model the division of two positive fractions and record the process symbolically.

i. Compare the division of positive fractions to the division of whole numbers, decimals, and integers.

j. Generalize and apply strategies for determining estimates of quotients of positive fractions.

k. Estimate the quotient of two given positive fractions and explain the strategy used.

l. Generalize and apply strategies for determining the quotients of positive fractions.

m. Critique the statement "Division always results in a smaller quantity" and reword the statement to capture the points of correction or clarification raised (e.g., $\frac{1}{2} \div \frac{1}{4} = 2$, but 2 is bigger than $\frac{1}{2}$ or $\frac{1}{4}$).

n. Identify, without calculating, the operation required to solve a problem involving fractions and justify the reasoning.

o. Create, represent (concretely, pictorially, or symbolically) and solve problems that involve one or more operations on positive fractions (including multiplication and division).

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***N8.5** Demonstrate understanding of multiplication and division of integers concretely, pictorially, and symbolically. [C, CN, PS, R, V]*	a. Identify and describe situations that are relevant to self, family, or community in which multiplication or division of integers would be involved. b. Model the multiplication of two integers using concrete materials or pictorial representations, and record the process used symbolically. c. Model the division of two integers using concrete materials or pictorial representations, and record the process used symbolically. d. Identify and generalize patterns for determining the sign of integer products and quotients. e. Generalize and apply strategies for multiplying and dividing integers. f. Create and solve problems involving the multiplication or division (without technology for one-digit divisors, with technology for two-digit divisors) of integers. g. Explain how the order of operations can be extended to include integers and provide examples to demonstrate the use of the order of operations. h. Create and solve problems requiring the use of the order of operations on integers.

Patterns and Relations Strand

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***P8.1** Demonstrate understanding of linear relations concretely, pictorially (including graphs), physically, and symbolically. [CN, ME, PS, R, T, V]*	a. Analyze and describe the relationship shown on a graph for a given situation (e.g., "The graph is showing that, as the temperature rises, the number of people in the mall decreases"). b. Explain how a given linear relation is represented by a given table of values. c. Model a linear relation shown as an equation, a graph, a table of values, or a concrete or pictorial representation in one or more other forms.

Outcomes	Indicators
P8.1 (continued)	d. Analyze a set of equations, graphs, ordered pairs, and tables of values, sort the set according to representing the same linear relations, and explain the reasoning. e. Determine the missing coordinate of an ordered pair given the equation of a linear relation, a table of values, or a graph and explain the reasoning. f. Determine which of a set of graphs, equations, tables of values, sets of ordered pairs, and concrete or pictorial representations represent a linear relationship and justify the reasoning. g. Determine if an ordered pair satisfies a linear relation given as a table of values, concrete or pictorial representation, graph, or equation and explain the reasoning. h. Identify situations relevant to self, family, or community that appear to define linear relations and determine, with justification, whether the graph for the situation would be shown with a solid line or not.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
P8.2 Model and solve problems using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b,\ a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c,\ a \neq 0$ • $a(x + b) = c$ ***concretely, pictorially, and symbolically, where a, b, and c are integers.*** ***[C, CN, PS, V]***	a. Identify and describe situations, which are relevant to self, family, or community, that can be modeled by a linear equation (e.g., the cost of purchasing x fish from a fisherman). b. Model and solve linear equations using concrete materials (e.g., counters and integer tiles) and describe the process orally and symbolically. c. Discuss the importance of the preservation of equality when solving equations. d. Explain the meaning of and verify the solution of a given linear equation using a variety of methods, including concrete materials, diagrams, and substitution. e. Generalize and apply symbolic strategies for solving linear equations. f. Identify, explain, and correct errors in a given solution of a linear equation. g. Demonstrate the application of the distributive property in the solving of linear equations (e.g., $2(x + 3);\ 2x + 6 = 5$)

Outcomes	Indicators
P8.2 (continued)	h. Explain why some linear relations (e.g., $\frac{x}{a} = b,\ a \neq 0$ and $\frac{x}{a} + b = c,\ a \neq 0$) have a given restriction and provide an example of a situation in which such a restriction would be necessary. i. Identify and solve problems that can be represented using linear equations and explain the meaning of the solution in the context of the problem. j. Explain the algebra behind a particular algebra puzzle such as this puzzle written for 2008: • Pick the number of times a week that you would like to go out to eat (more than once but less than 10). • Multiply this number by 2 (just to be bold). • Add 5. • Multiply it by 50. • If you have already had your birthday this year add 1758. If you have not, add 1757. • Now subtract the four digit year that you were born. • You should have a three digit number. The first digit of this was your original number. The next two numbers are your age.

Shape and Space Strand

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS8.1** **Demonstrate understanding of the Pythagorean Theorem concretely or pictorially and symbolically and by solving problems. ***[CN, PS, R, T, V]***	a. Generalize the results of an investigation of the expression $a^2 + b^2 = c^2$ (where a, b, and c are the lengths of the sides of a right triangle, c being the longest): • concretely (by cutting up areas represented by a^2 and b^2 and fitting the two areas onto c^2) • pictorially (by using technology) • symbolically (by confirming that $a^2 + b^2 = c^2$ for a right triangle). b. Explore right and non-right triangles, using technology, and generalize the relationship between the type of triangle and the Pythagorean Theorem (i.e., if the sides of a triangle satisfy the Pythagorean equation, then the triangle is a right triangle which is known as the Converse of the Pythagorean Theorem).

Outcomes	Indicators
SS8.1 (continued)	c. Explore right triangles, using technology, using the Pythagorean Theorem to identify Pythagorean triples (e.g., 3, 4, 5 or 5, 12, 13), hypothesize about the nature of triangles with side lengths that are multiples of the Pythagorean triples, and verify the hypothesis. d. Create and solve problems involving the Pythagorean Theorem, Pythagorean triples, or the Converse of the Pythagorean Theorem. e. Give a presentation that explains a historical or personal use or story of the Pythagorean Theorem (e.g., Pythagoras and his denial of irrational numbers, the use of the 3:4:5 right triangle ratio in the Pyramids, squaring off the corner of a sandbox being built for a sibling, or determining the straight line distance between two towns to be travelled on a snowmobile).

Goals: Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
***SS8.2** Demonstrate understanding of the surface area of 3-D objects limited to right prisms and cylinders (concretely, pictorially, and symbolically) by:* • *analyzing views* • *sketching and constructing 3-D objects, nets, and top, side, and front views* • *generalizing strategies and formulae* • *analyzing the effect of orientation* • *solving problems.* *[C, CN, PS, R, T V]*	a. Manipulate concrete 3-D objects to identify, describe, and sketch top, front, and side views of the 3-D object on isometric paper. b. Sketch a top, front, or side view of a 3-D object that is within the classroom or that is personally relevant, and ask a peer to identify the 3-D object it represents. c. Predict the top, front, and side views for a 3-D object that is to be rotated by a multiple of 90°, discuss the reasoning for the prediction, and then verify concretely and pictorially. d. Identify and describe nets of 3-D objects that are used in everyday experiences (e.g., such as patterns or materials for clothing and banker boxes). e. Relate the parts (using one-to-one correspondence) of a net to the faces and edges of the 3-D object it represents. f. Create a net for a 3-D object, have a peer predict the type of 3-D object that the net represents, explain to the peer the reasoning used in designing the net, and have the peer verify the net by constructing the 3-D object from the net. g. Build a 3-D object made of right rectangular prisms based on the top, front, and side views (with and without the use of technology).

Outcomes	Indicators
SS8.2 (continued)	h. Demonstrate how the net of a 3-D object (including right rectangular prisms, right triangular prisms, and cylinders) can be used to determine the surface area of the 3-D object and describe strategies used to determine the surface area. i. Generalize and apply strategies for determining the surface area of 3-D objects. j. Create and solve personally relevant problems involving the surface area or nets of 3-D objects.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS8.3** **Demonstrate understanding of volume limited to right prisms and cylinders (concretely, pictorially, or symbolically) by: • ***relating area to volume*** • ***generalizing strategies and formulae*** • ***analyzing the effect of orientation*** • ***solving problems.*** *[CN, PS, R, V]*	a. Identify situations from one's home, school, or community in which the volume of right prism or right cylinder would need to be determined. b. Describe the relationship between the area of the base of a right prism or right cylinder and the volume of the 3-D object. c. Generalize and apply formulas for determining the area of a right prism and right cylinder. d. Explain the effect of changing the orientation of a right prism or right cylinder on the volume of the 3-D object. e. Create and solve personally relevant problems involving the volume of right prisms and right cylinders.

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS8.4** **Demonstrate an understanding of tessellation by: • ***explaining the properties of shapes that make tessellating possible*** • ***creating tessellations*** • ***identifying tessellations in the environment.*** *[C, CN, PS, T, V]*	a. Identify, describe (in terms of translations, reflections, rotations, and combinations of any of the three), and reproduce (concretely or pictorially) a tessellation that is relevant to self, family, or community (e.g., a Star Blanket or wall paper). b. Predict and verify which of a given set of 2-D shapes (regular and irregular) will tessellate and generalize strategies for determining whether a new 2-D shape will tessellate (i.e., an angle must be a factor of 360°).

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SS8.4 (continued)	c. Identify one or more 2-D shapes that will tessellate with a given 2-D shape and explain the choice (e.g., knowing that the sum of the measures of one angle from each of the 2-D shapes must be a factor of 360°, and if the given shape has an angle of 12°, then two shapes with angles of 13° and 5° can be used to tessellate with the original shape because 12+13+5=30 which is a factor of 360 – these shapes would need to be repeated at least 12 times because 30 x 12 is 360). d. Design and create (concretely or pictorially) a tessellation involving one or more 2-D shapes, and document the mathematics involved within the tessellation (e.g., types of transformations, measures of angles, or types of shapes). e. Identify different transformations (translations, reflections, rotations, and combinations of any of the three) present within a tessellation. f. Make a new tessellating shape (polygonal or non-polygonal) by transforming a portion of a known tessellating shape and use the new shape to create an Escher-type design that can be used as a picture or wrapping paper.

Statistics and Probability Strand

Goals: Number Sense, Spatial Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SP8.1 Analyze the modes of displaying data and the reasonableness of conclusions. [C, CN, R]	a. Investigate and report on the advantages and disadvantages of different types of graphs, including circle graphs, line graphs, bar graphs, double bar graphs, and pictographs (e.g., circle graphs are good for qualitative data such as favourite activities and categories such as money spent on clothes, whereas line graphs are good for quantitative data such as heights and ages b. Engage in a project that involves: • the collection and organization of first- or second-hand data related to a topic of interest (such as local wildlife counts or surveying of peers) • representation of the data using a graph • explanation of type of graph chosen by self and peer • description of the project, challenges, and conclusions • self-assessment.

Outcomes	Indicators
SP8.1 (continued)	c. Suggest alternative ways to represent data from a given situation and explain the choices made. d. Find examples of graphs of data in media and personal experiences and interpret the information in the graphs for personal value. e. Analyze a data graph found in media for features that might bias the interpretation of the graph (such as the size of intervals, the width of bars, and the visual representation) and suggest alterations to remove or downplay the bias. f. Provide examples of misrepresentations of data and data graphs found within different media and explain what types of misinterpretations might result from such displays.

Goals: Number Sense, Logical Thinking, Mathematics as a Human Endeavour

Outcomes	Indicators
SP8.2 Demonstrate understanding of the probability of independent events concretely, pictorially, orally, and symbolically. *[C, CN, PS, T]*	a. Ask questions relevant to self, family, or community in which probabilities involving two events are known or which can be researched. b. Explore and explain the relationship between the probability of two independent events and the probability of each event separately. c. Make and test predictions about the results of experiments and simulations for two independent events. d. Create and solve problems related to independent events, probabilities of independent events, and decision making.

Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement must be based on the outcomes in the provincial curriculum.

Assessment involves the systematic collection of information about student learning with respect to:

- ☑ Achievement of provincial curriculum outcomes
- ☑ Effectiveness of teaching strategies employed
- ☑ Student self-reflection on learning.

Assembling evidence from a variety of sources is more likely to yield an accurate picture.
(NCTM, 2000, p. 24)

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Reporting of student achievement must be based on the achievement of curriculum outcomes. Assessment information which is not related to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Grade 8 Mathematics. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

Assessment should not merely be done to students; rather it should be done for students.
(NCTM, 2000, p. 22)

Assessment as learning involves student reflection on and monitoring of her/his own progress and:

- students self-reflect and critically analyze learning related to curricular outcomes without anxiety or censure
- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

What are examples of assessments as learning that could be used in Grade 8 Mathematics and what would be the purpose of those assessments?

Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement and:

- provides opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussions on placement or promotion.

Assessment should become a routine part of the ongoing classroom activity rather than an interruption.
(NCTM, 2000, p. 23)

In mathematics, students need to be regularly engaged in assessment as learning. The assessments used should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for the students to set and assess personal learning goals related to the mathematical content for Grade 8.

Connections with Other Areas of Study

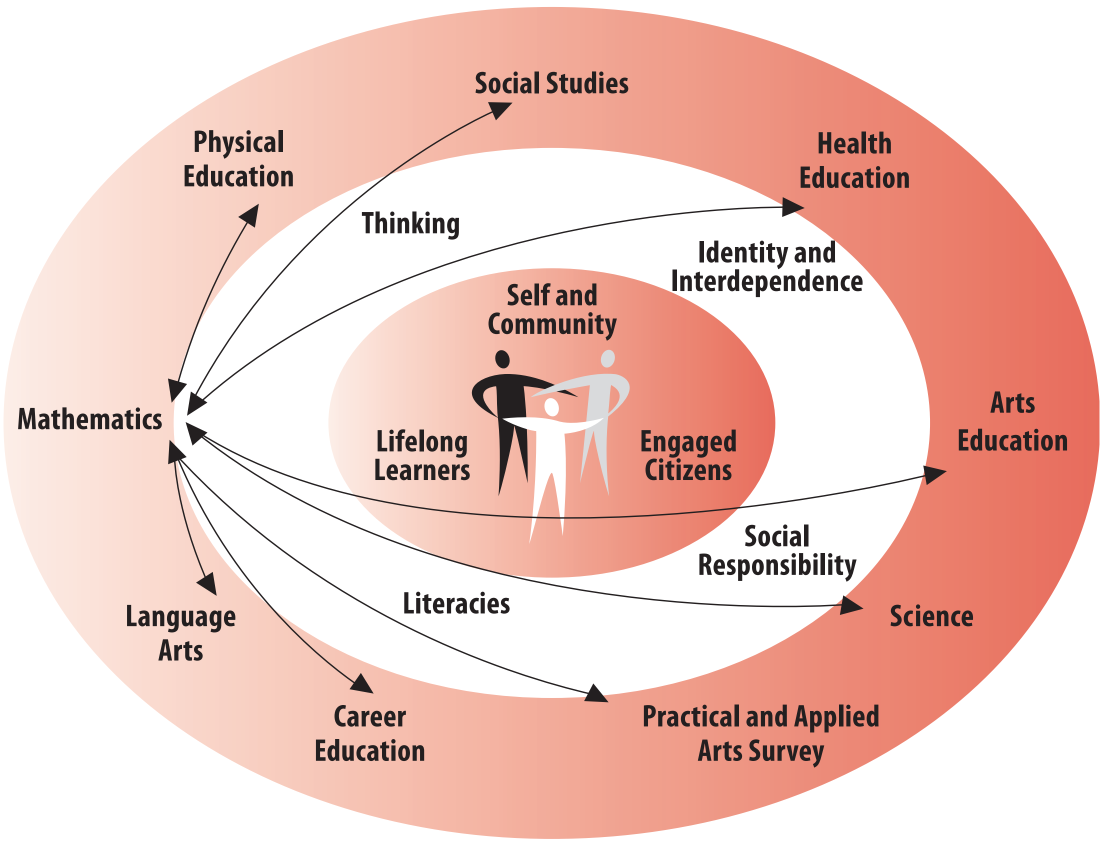
There are many possibilities for connecting Grade 8 mathematical learning with the learning occurring in other subject areas. When making such connections, however, teachers must be cautious not to lose the integrity of the learning in any of the subjects. Making connections between subject areas gives students experiences with transferring knowledge and provides rich contexts in which students are able to initiate, make sense of, and extend their learnings. When connections between subject areas are made, the possibilities for transdisciplinary inquiries and deeper understanding arise. Following are just a few of the ways in which mathematics can be connected to other subject areas (and other subject areas connected to mathematics) at Grade 8.

Arts Education – As the complexity of the abstraction in mathematics increases, the understandings that students learn in Arts Education can serve as an effective means for students to communicate their mathematical knowledge. For example, in conveying their understanding of tessellations, outcome SS8.4:

> Demonstrate an understanding of tessellations by:
> - explaining the properties of shapes that make tessellating possible
> - creating tessellations
> - identifying tessellations in the environment.
>
> [C, CN, PS, T, V]

students might create a dance to demonstrate different transformations and relationships between angles within tessellations.

The students can also use their understanding of tessellations to create art works that integrate different elements of art and principles of composition such as rhythm and pattern. In addition, art work that students create and critique in Arts Education can be analyzed for evidence of different transformations and tessellations.



Career Education – In Grade 8 Career Education, students are exploring how their personal behaviours and attitudes influence their self-image, as well as life and work. These explorations can be connected to the students' study of mathematics through outcome SP8.1:

Analyze the modes of displaying data and the reasonableness of conclusions. [C, CN, R]

Students can collect first-hand and second-hand data about their personal behaviours and attitudes and how those behaviours and attitudes influence self-image, life, and work. Using this data, the students can create statistical displays, justifying the choice of display used, and then analyze the representation to identify challenges and conclusions that can be reached. Students can extend this activity further by creating an action plan for one or more changes to their personal behaviours and attitudes, and chart and analyze their progress mathematically. In doing this exploration, students may also be connecting with their learnings from Health Education, Physical Education, and English Language Arts.

English Language Arts (ELA) – ELA and Mathematics share a common interest in students developing their abilities to reflect upon and communicate about their learnings through viewing, listening, reading, representing, speaking, and writing. As an example of how Mathematics involves these strands of language consider outcome SP8.1:

> Analyze the modes of displaying data and the reasonableness of conclusions. [C, CN, ME, PS, R, V]

Part of the analysis of the modes of displaying data requires the students to view displays of data and to critique the use of those displays related to bias and effectiveness. In achieving this outcome, students will also need to communicate with each other and their teacher through dialogue and the use of representations. This communication will require students to engage in strategies and skills from ELA related listening, reading, speaking, representing, and writing.

In addition, to successfully attain outcome SP8.1, students will need to engage in reading informational texts with comprehension and communicate the meaning gained from these texts to other data. Both the reading of informational texts and the communication of meaning are part of the outcomes in Grade 8 ELA.

Connections can also be made between outcomes N8.1 and SS8.1:

> Demonstrate understanding of the square and principle square root of whole numbers concretely or pictorially and symbolically. [CN, ME, R, T, V]
>
> Demonstrate understanding of the Pythagorean Theorem concretely or pictorially and symbolically and by solving problems. [CN, PS, R, T, V]

and ELA's study of narratives of Greek peoples. As students explore these narratives, students are introduced to texts related to Pythagoras and his time period. This connection will help students gain understandings for the Mathematics as a Human Endeavour goal by showing that mathematics has been developed by people and that, like other human developments, it was influenced by the time in which it occurred. Moreover, the students can learn that the development of mathematics was often rich in intrigue and scandal.

Health Education – In Grade 8, the strongest connections between Health Education and Mathematics are related to outcome SP8.1:

> Analyze the modes of displaying data and the reasonableness of conclusions.

In Grade 8 Health Education, students are examining and assessing the impacts and importance of a number of concepts related to health and well-being such as sustainability, supports, and services available related to non-curable infections and diseases, and personal prejudices and biases. Much of the data that students will collect and find through research will be presented using a variety of data displays that students are familiar with from their study of mathematics. These data sets and displays and the reasonableness of the conclusions from a Health Education perspective can be critiqued by the students using strategies developed in Mathematics class.

Physical Education – In Grade 8 Physical Education, there are many opportunities for connections to Grade 8 Mathematics. Most notable are the students' personal plans for improving skill-related components of fitness. Part of creating and carrying out a personal improvement plan is the tracking and analysis of progress. This aspect of the student's plan can be achieved through the mathematics outcome SP8.1:

> Analyze the modes of displaying data and the reasonableness of conclusions. [C, CN, R]

Data collected in the students' Physical Education classes related to their personal improvement plans can be analyzed through the creation of different data representations and interpretation of those displays. As part of their learning, students could be asked to draw and justify conclusions about their progress based upon their analysis of the data in Mathematics.

Science – In Grade 8 Science, there are many opportunities for students to make connections with outcome SP8.1:

> Analyze the modes of displaying data and the reasonableness of conclusions. [C, CN, R]

When students evaluate the impact of electromagnetic radiation based technologies, explore ethical issues related to the use of technologies for medical applications, and analyze and synthesize information regarding the ways we should maintain our body, students will need to analyze the source, presentation, and data obtained. Students can use their mathematical knowledge to determine the reasonableness of the conclusions reached and can combine their mathematical and scientific understandings to make decisions regarding students' own beliefs and conclusions.

Social Studies – Grade 8 Social Studies provides many rich contexts which can be used in students' study of outcome SP8.1:

> Analyze the modes of displaying data and the reasonableness of conclusions. [C, CN, R]

Many of the topics within the Grade 8 Social Studies outcomes will introduce the students to a variety of reports of data, some of which may be conflicting. Students will need to interpret and assess the reasonableness of conclusions. For example, the students' study of how historical events have affected the present Canadian identity, and what the present Canadian identity is, could bring to light many statistical displays and conclusions.

Glossary

Benchmarks: Numeric quantities used to compare and order other numeric quantities. For example, 0, 5, 10, and 20 are often used as benchmarks when placing whole numbers on a number line.

Equality as a Balance and Inequality as Imbalance: The equal sign represents the idea of equivalence. For many students, it means *do the question*. For some students, the equal sign in an expression such as 2 + 5 = means to *add*. When exploring equality and inequality, by using objects on a balance scale, students discover the relationships between and among the mass of the objects. The equal sign in an equation is like a scale: both sides, left and right, must be the same in order for the scale to stay in balance and the equation to be true. When the scale is imbalanced, the equation is not true. Using 2 + 5 =□ , rather than simply 2 + 5 = helps students understand that the equal sign (=) represents equality rather than *"do the work"* or *"do the question"*.

Independent Events: Two events are independent if the occurrence of one of the events gives no information about whether or not the other will occur; that is, the events have no influence on each other. For example, the type of sandwich someone orders and the type of drink ordered are independent events because the ordering of the sandwich does not influence the drink being ordered or vice versa.

Interdisciplinary: Disciplines connected by common concepts and skills embedded in disciplinary outcomes.

Linear Relation: A mathematical equation that represents a relationship between two variables whose graph is a straight line. For example: $y = 3x - 8$ is a linear relation.

Multidisciplinary: Discipline outcomes organized around a theme and learned through the structure of the disciplines.

Number, Numeral, Digit: A number is the name that we give to quantities. For example, there are 7 days in a week, or I have three brothers – both seven and three are numbers in these situations because they are defining a quantity. The symbolic representation of a number, such as 287, is called the numeral. If 287 is not being used to define a quantity, we call it a numeral.

Numerals, as the symbolic representation of numbers, are made up of a series of digits. The Hindu-Arabic number system that we use has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. (Note: sometimes students are confused between these digits and their finger digits – this is because they count their fingers starting at one and get to ten rather than zero to nine.) These digits are also numerals and can be numbers (representing a quantity), but all numbers and all numerals are combinations of digits. The placement of a digit in a number or numeral affects the place value of the digit and, hence how much of the quantity that it represents. For example, in 326, the 2 is contributing 20 to the total, while in 236 the 2 contributes 200 to the total quantity.

Object: Object is used to refer to a three-dimensional geometrical figure. To distinguish this meaning from that of shape, the word "object" is preceded by the descriptor "3-D".

Personal Strategies: Personal strategies are strategies that the students have constructed and understand. Outcomes and indicators that specify the use of personal strategies convey the message that there is not a single procedure that is correct. Students should be encouraged to explore, share, and make decisions about what strategies to use in different contexts. Development of personal strategies is an indicator of the attainment of deeper understanding.

Principle Square Root: The principle square root of a number is the positive quantity that, when squared, gives the original number. The principle square root of a number is shown symbolically as $\sqrt{a}$ or $+\sqrt{a}$.

Proportional Reasoning: Two quantities are proportional if the ratio or rate that compares the two is constant. Proportional reasoning is using this property to determine new quantities in new situations.

Rate: A comparison of two measurements with different units. For example, $3/kg is a rate.

Ratio: A comparison of two quantities with the same units so the units are not required. For example, if 45 mL of syrup is to be mixed with 1 L of water, the ratio is 45:1000 (read 45 to 1000). Ratios can also be written in fractional form ($\frac{45}{1000}$), but it is important to note that a ratio can be a comparison of part-to-part as well as part-to-whole, where a quantity referred to as a fraction is related to a part-to-whole comparison only.

Representations: Mathematical ideas can be represented and manipulated in a variety of forms including concrete manipulatives, visual designs, sounds and speech, physical movements, and symbolic notations (such as numerals and operation signs). Students need to have experiences in working with many different types of representations, and in transferring and translating knowledge between the different forms of representations.

Right Prisms and Cylinders: 3-D objects that are prisms or cylinders with the additional condition that the height of the object be perpendicular to its base.

Shape: In this curriculum, shape is used to refer to two-dimensional geometric figures and is thus preceded by "2-D". The term shape is sometimes also used in mathematics resources and conversations to refer to three-dimensional geometric figures. It is important that students learn to be clear in identifying whether their use of the term shape is in reference to a 2-D or 3-D geometrical figure.

Tessellation: A collection of 2-D shapes that fills a plane with no overlaps and gaps. Tessellations at the grade 8 level include translations, reflections, and rotations of the 2-D shapes.

Transdisciplinary: All knowledge interconnected and interdependent; real-life contexts emphasized and investigated through student questions.

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